Region Select Game Walkthrough

using Knot Theory

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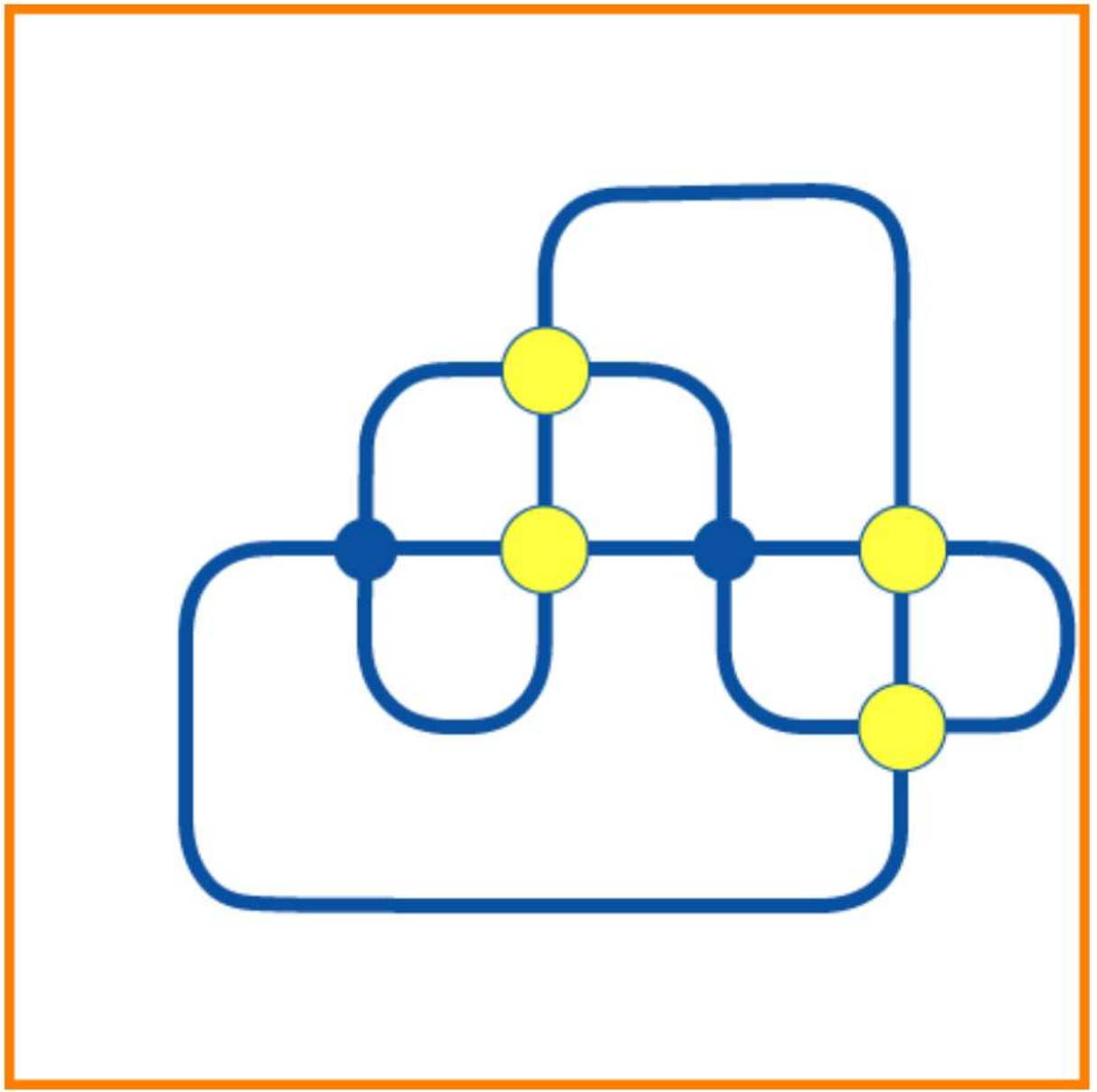
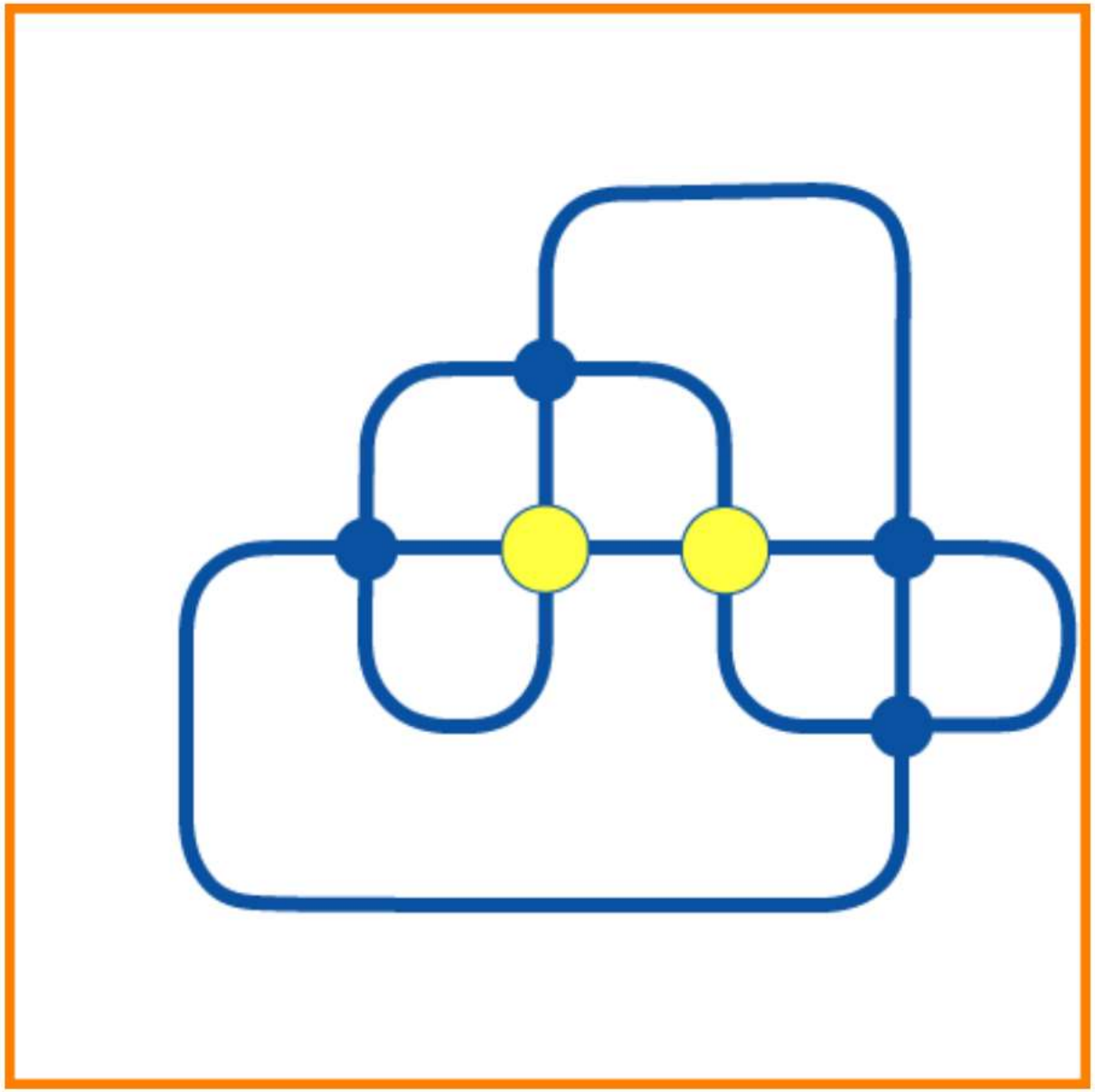
1. Region Select Game?

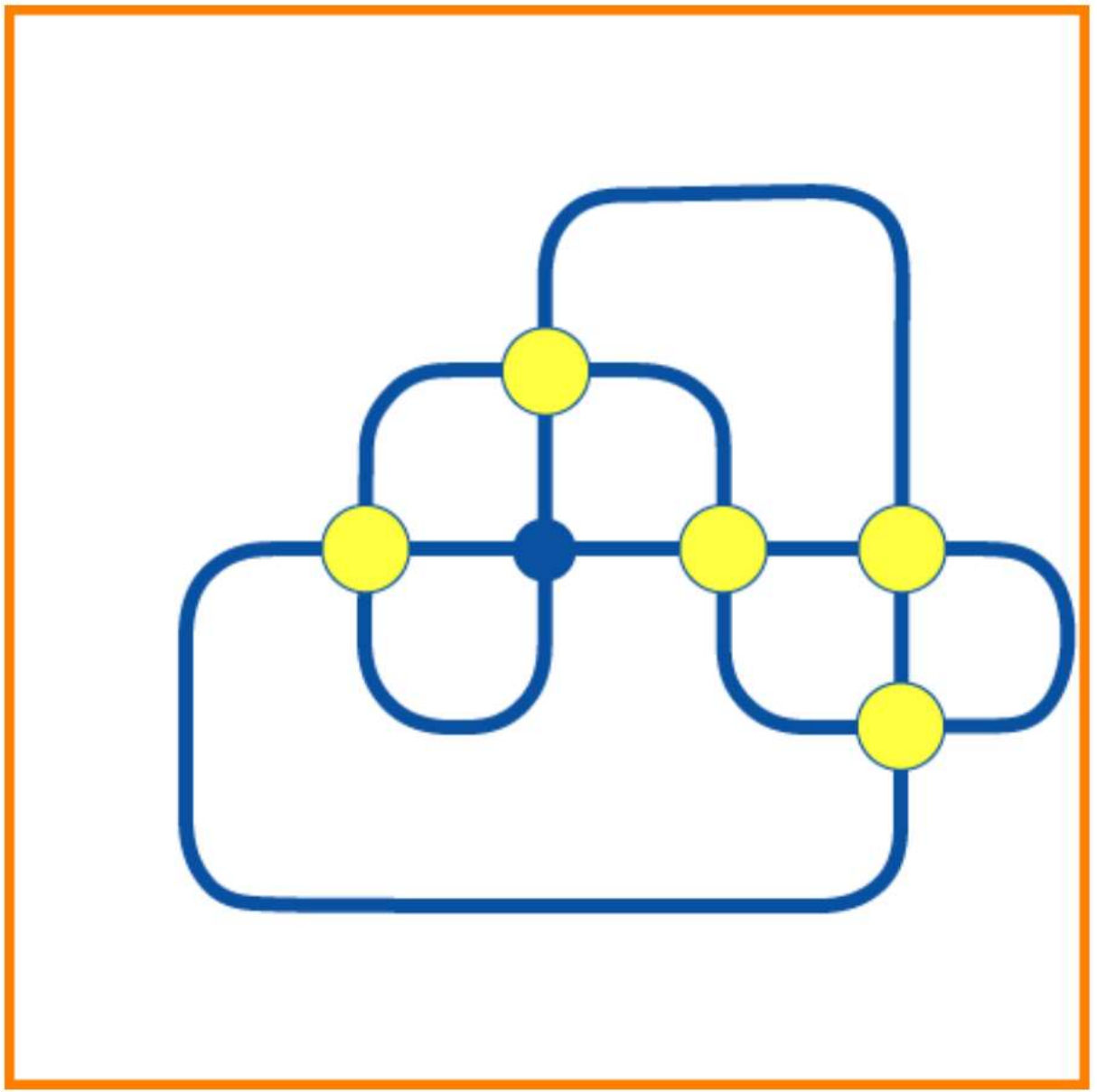
Do it!



Do it!



Do it!



Question.

어떤 상태로 시작하든 모든 전구를 다 킬 수 있을까?

어떤 전구를 하나만 킬 수 있는 알고리즘이 있을까?

2. Main Theorem

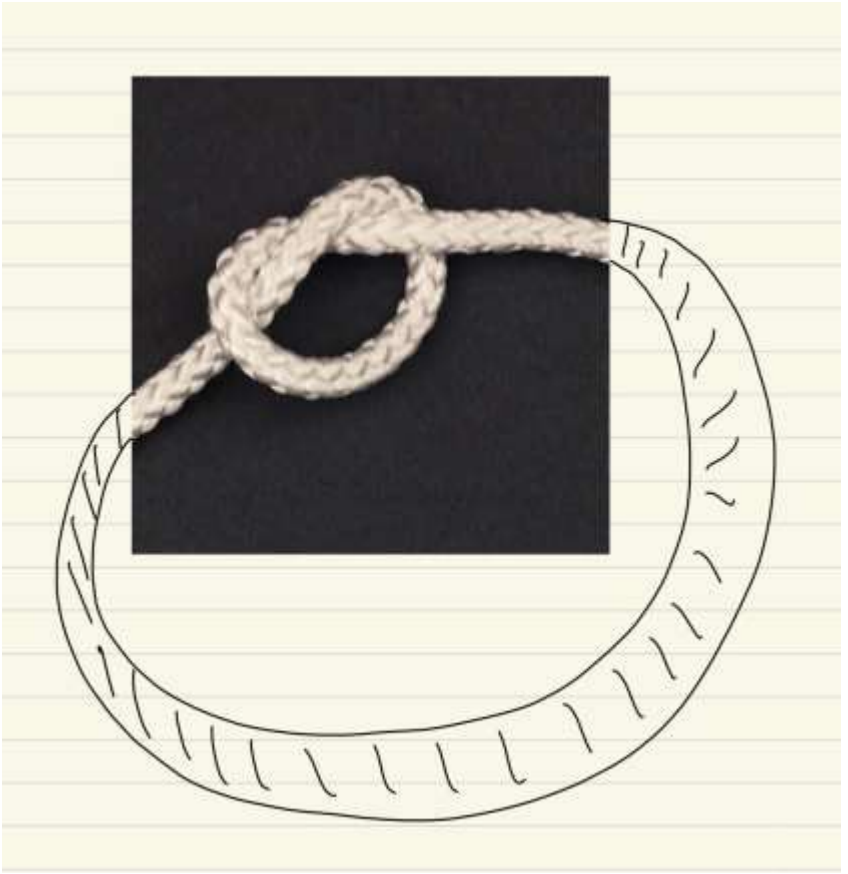
THEOREM 1.1. *Let D be a knot diagram, and c a crossing point of D. Let D' be the diagram obtained from D by the crossing change at c. Then, there exist region crossing changes which transform D into D'.*

Preliminaries

1. *knot diagram,*

So what is a knot?

Reference : https://en.wikipedia.org/wiki/Overhand_knot



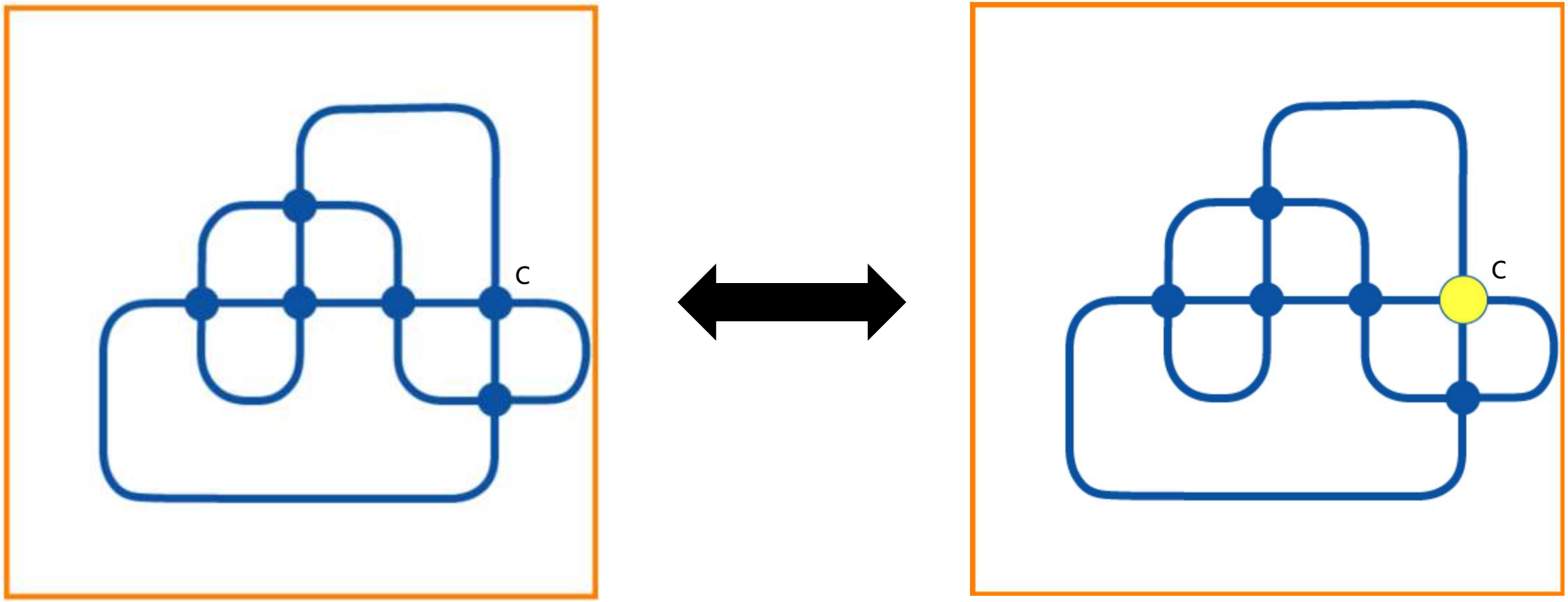
Preliminaries

2. *crossing point*

Knot diagram

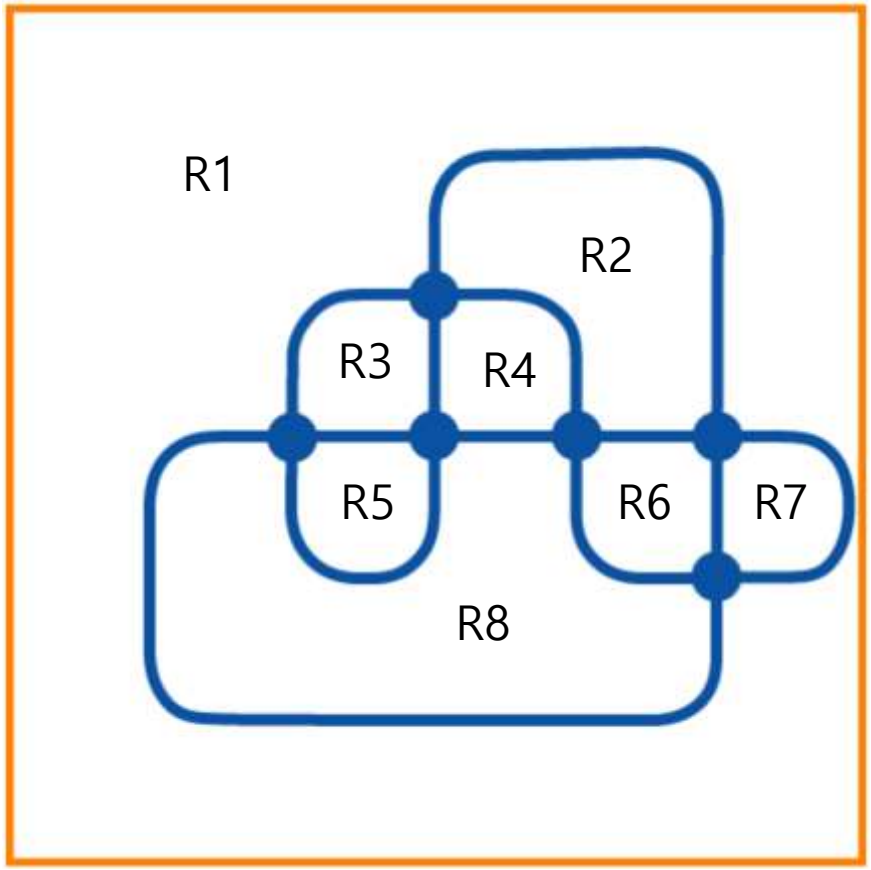
Preliminaries

3. *crossing change at c*



Preliminaries

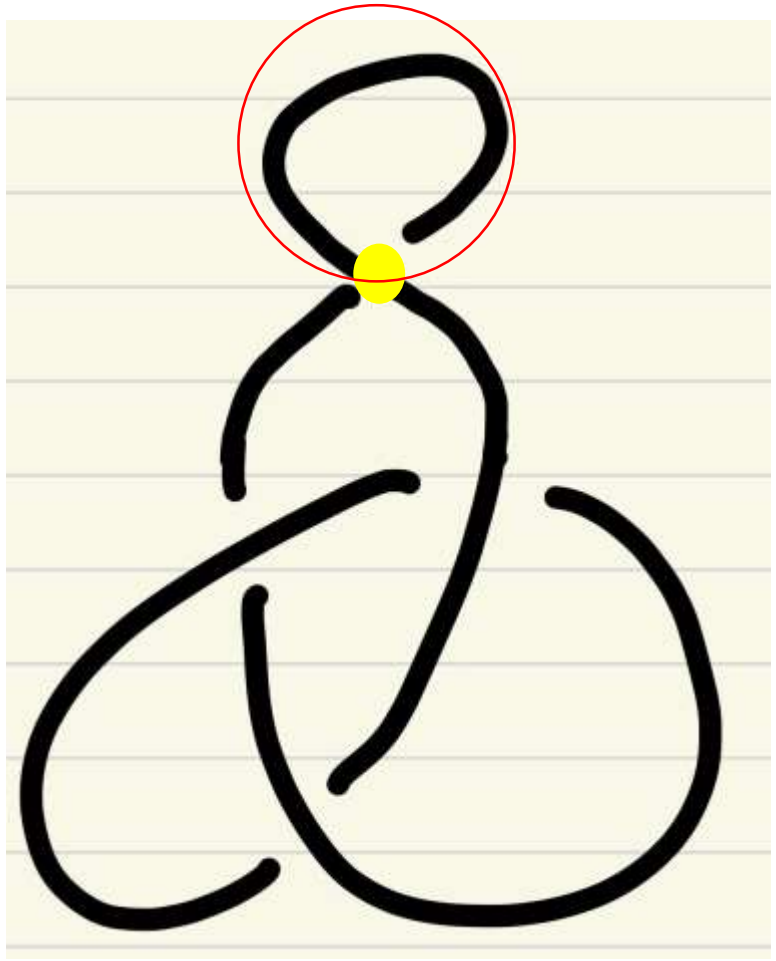
4. *region crossing changes*



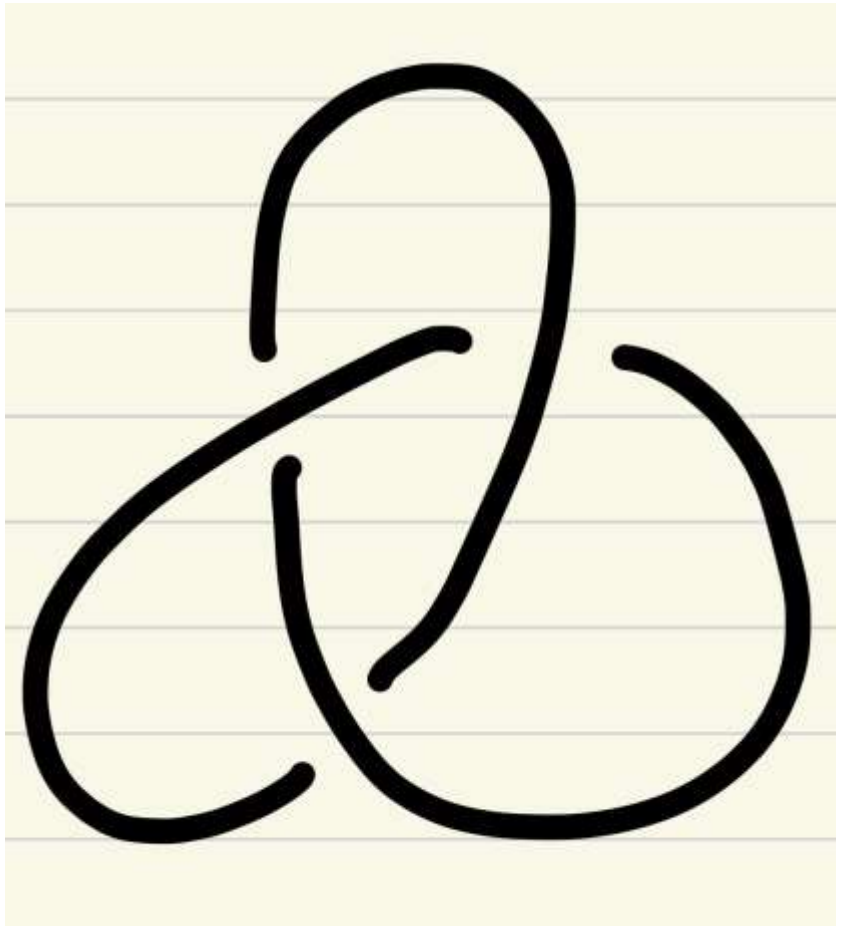
Preliminaries

5. Reduced, reducible

Reducible diagram



Reduced diagram



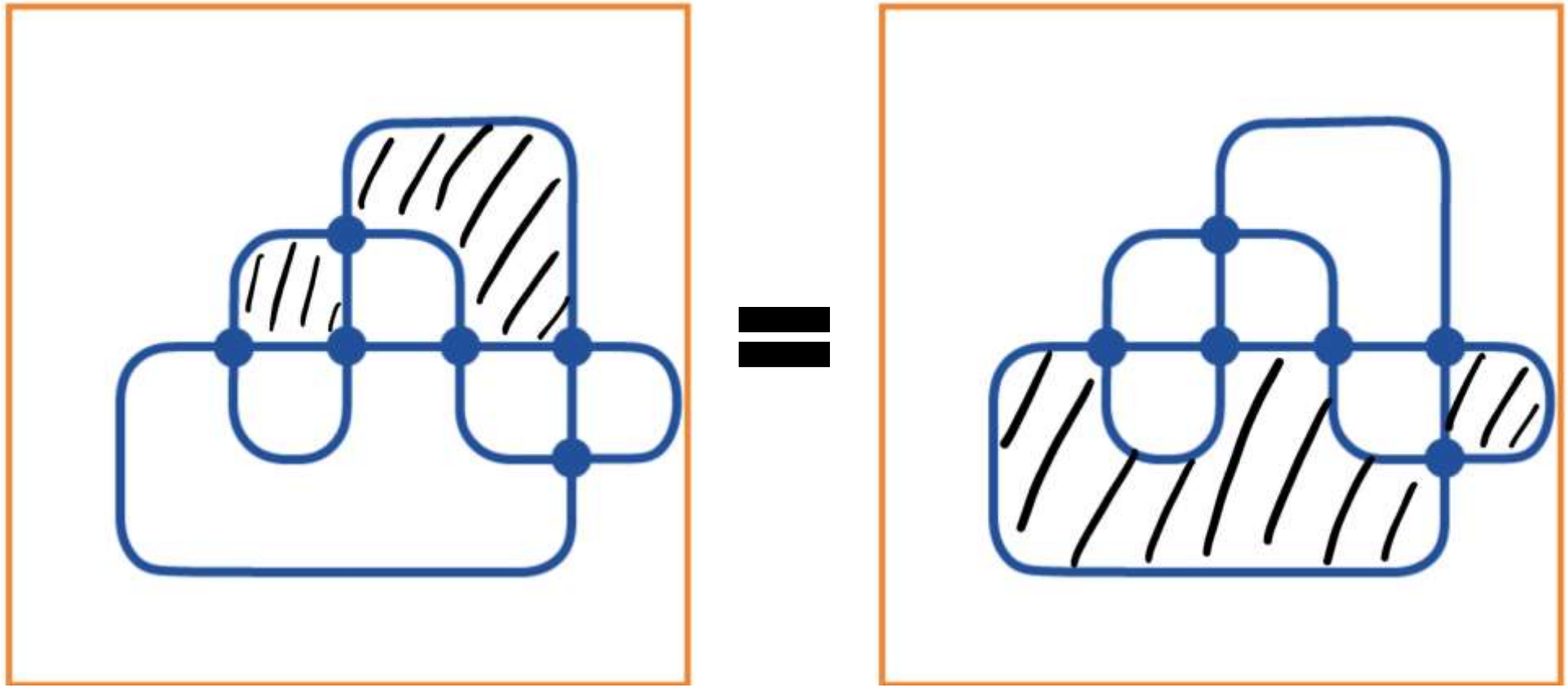
3. How to prove?

Corollary 1. 색칠한 곳에 전구를 다 켜도 원래 상태로 돌아온다.

Checkerboard coloring

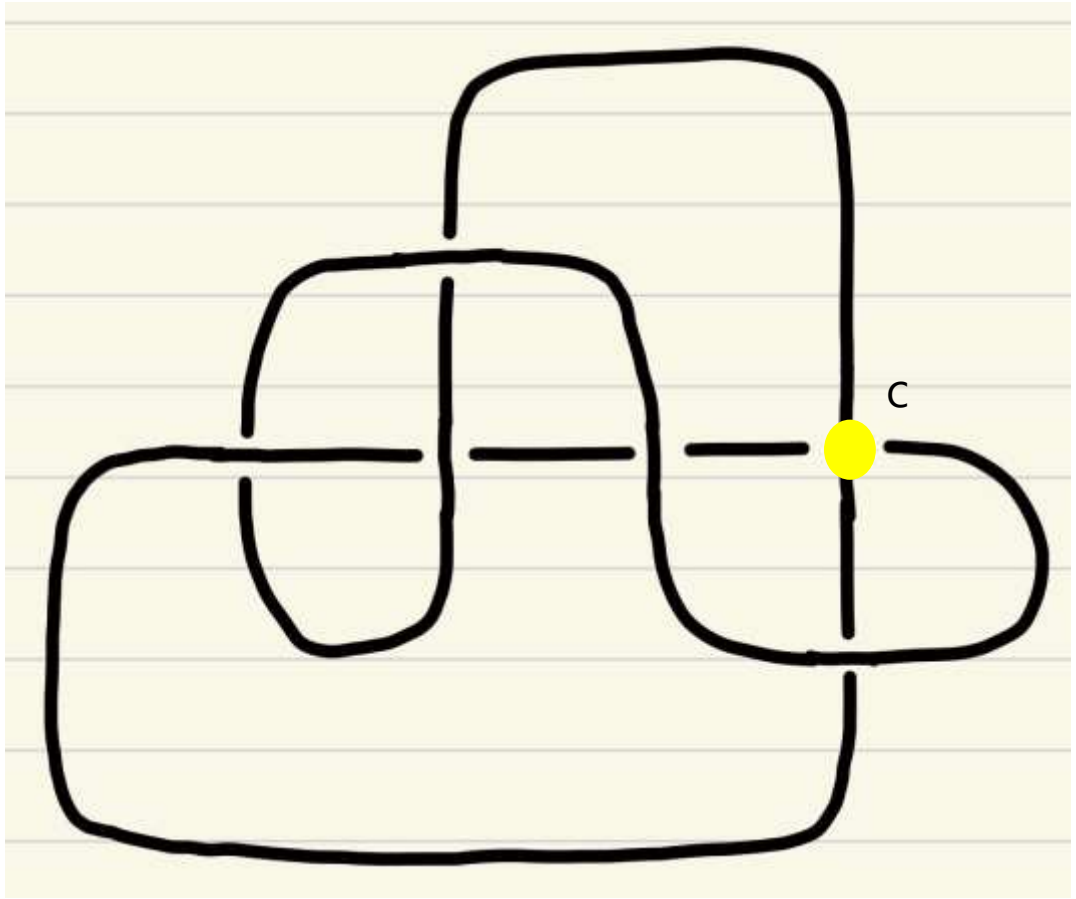
3. How to prove?

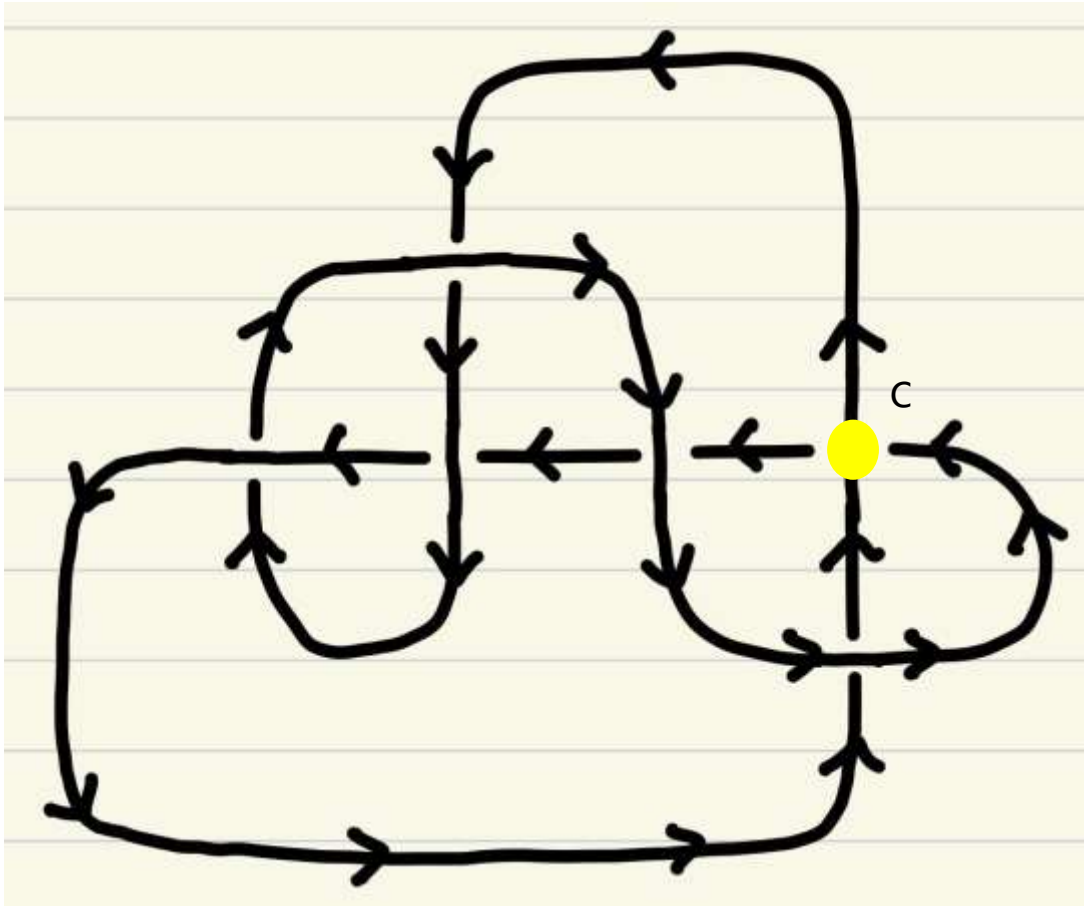
Corollary 2. 색칠한 곳에 전구를 일부분 킨 것과 나머지 부분을 킨 것의 상태와 같다.



3. How to prove?

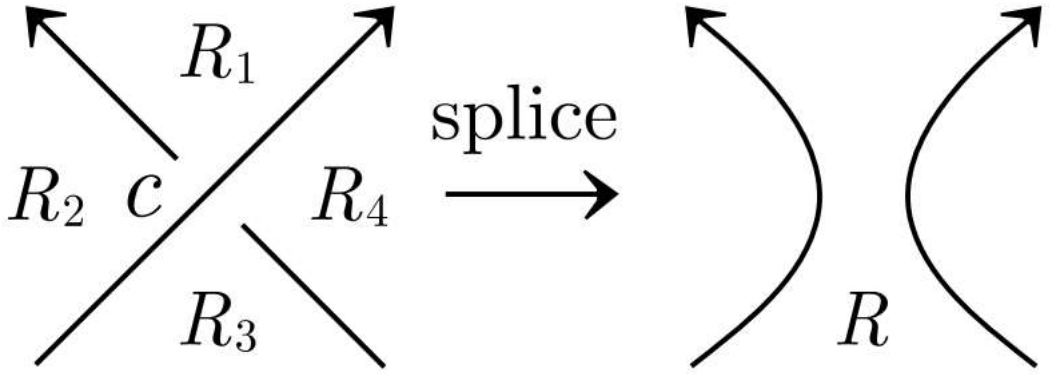
Step 1.

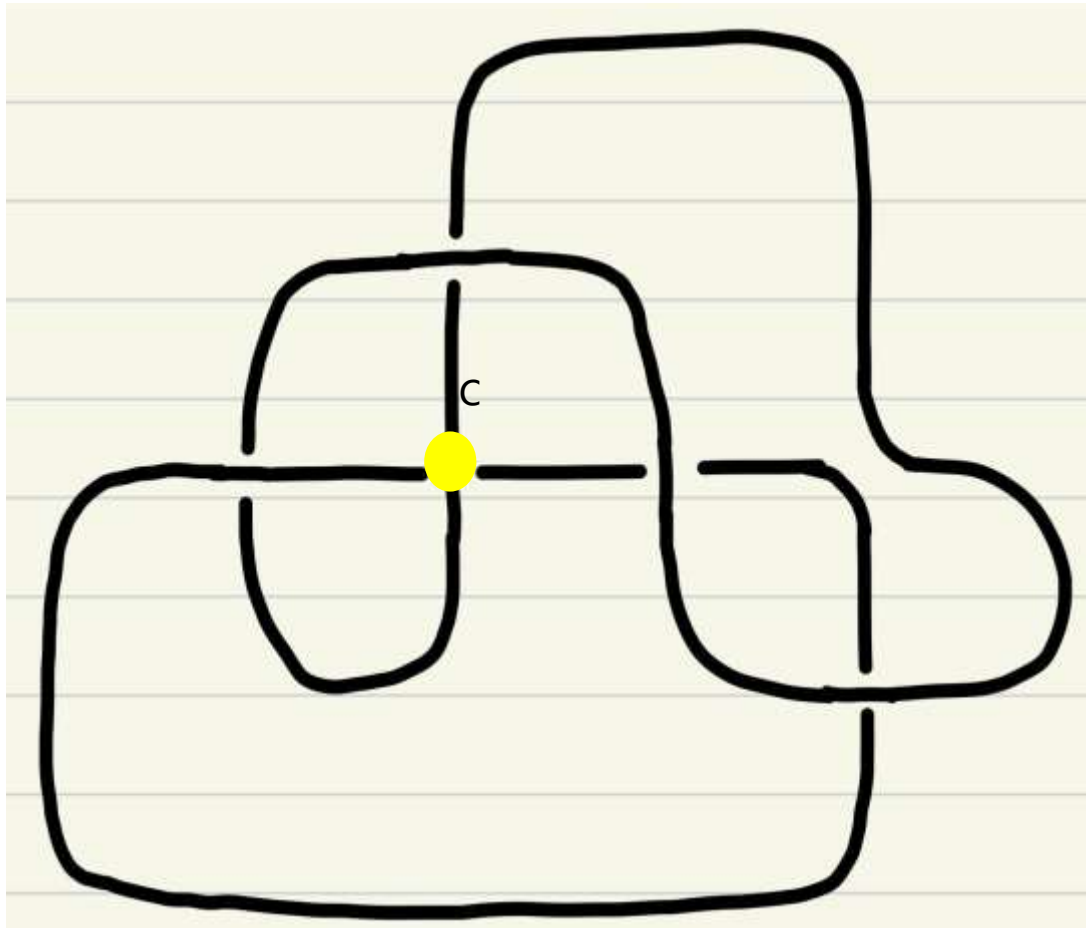




3. How to prove?

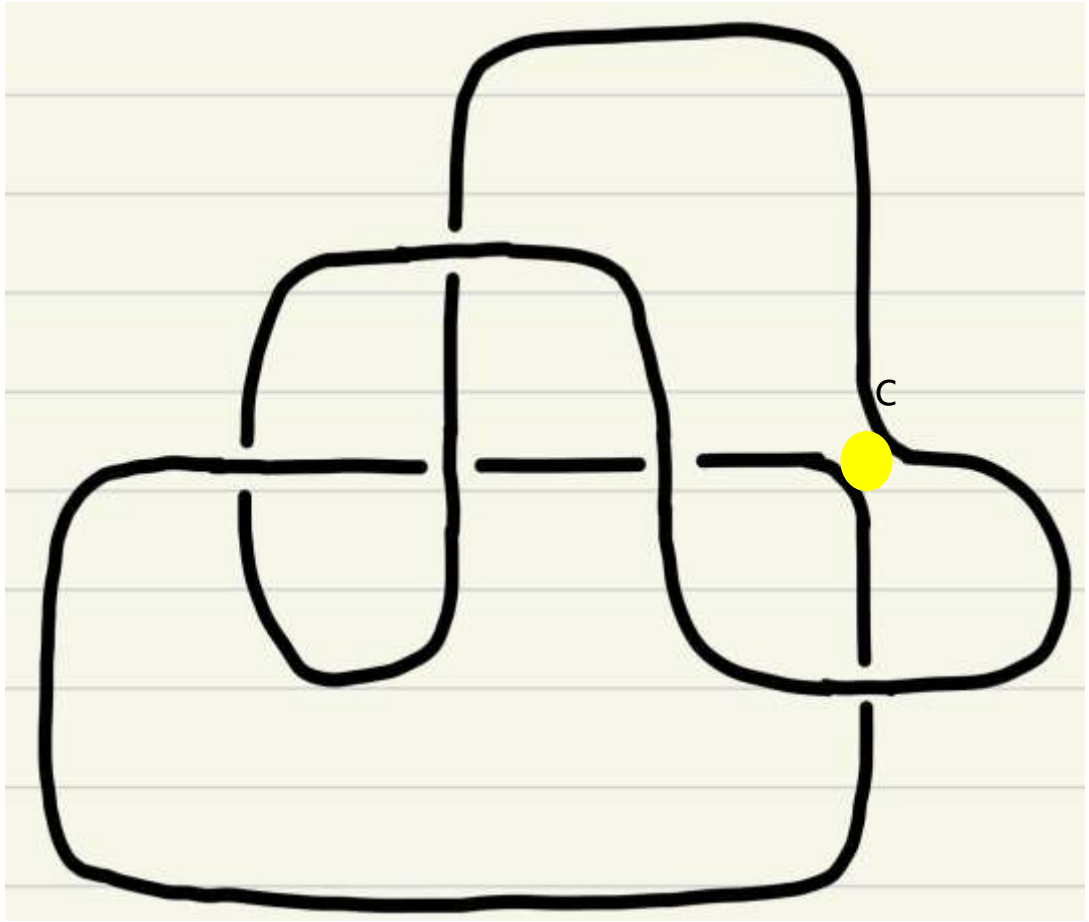
Step 2.

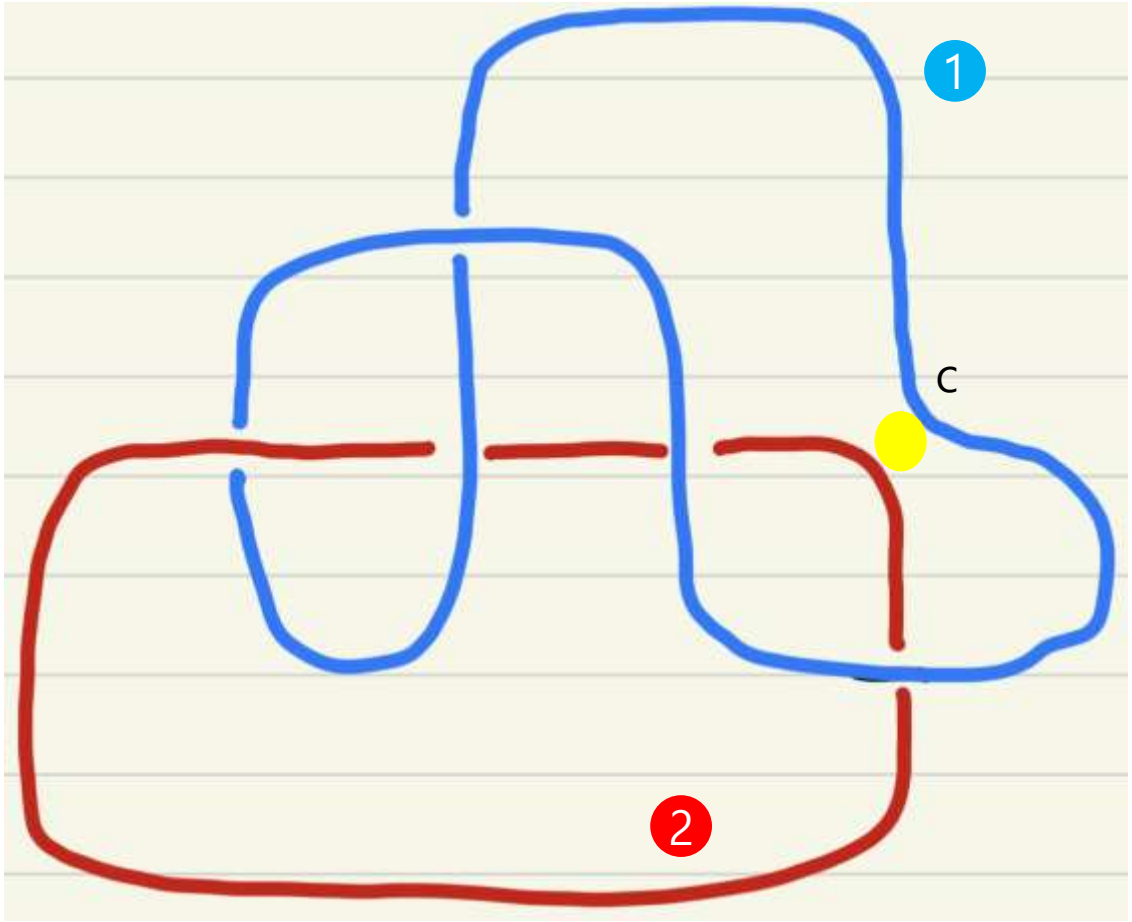




3. How to prove?

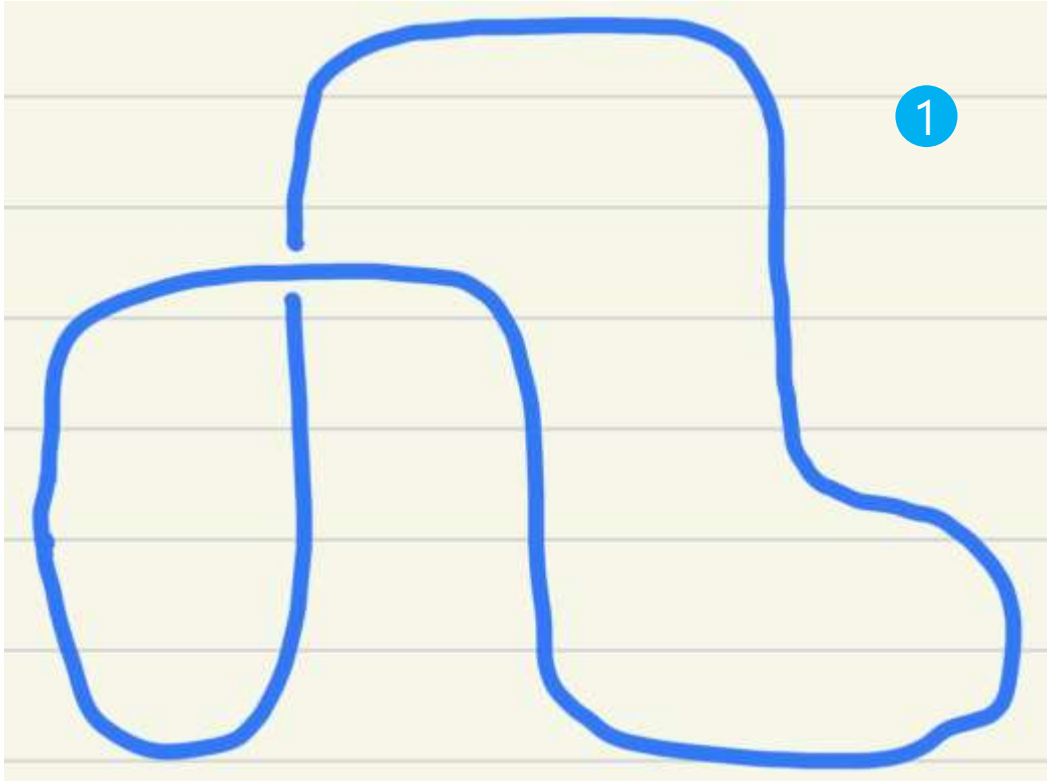
Step 3.

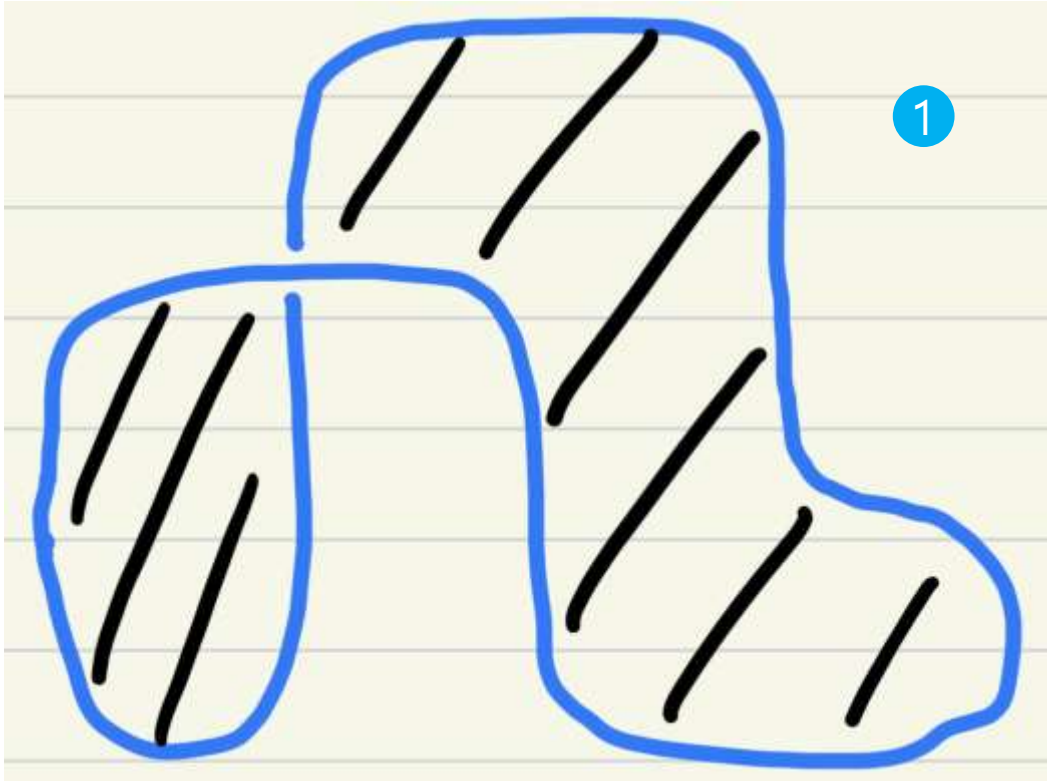




3. How to prove?

Step 4. Choose one component and checkerboard coloring.

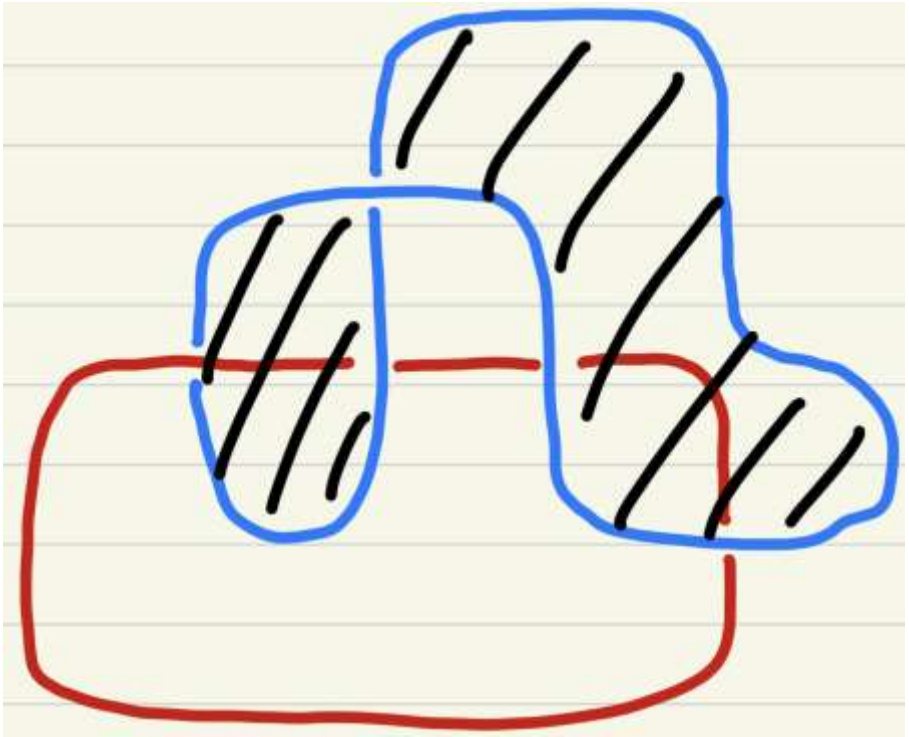


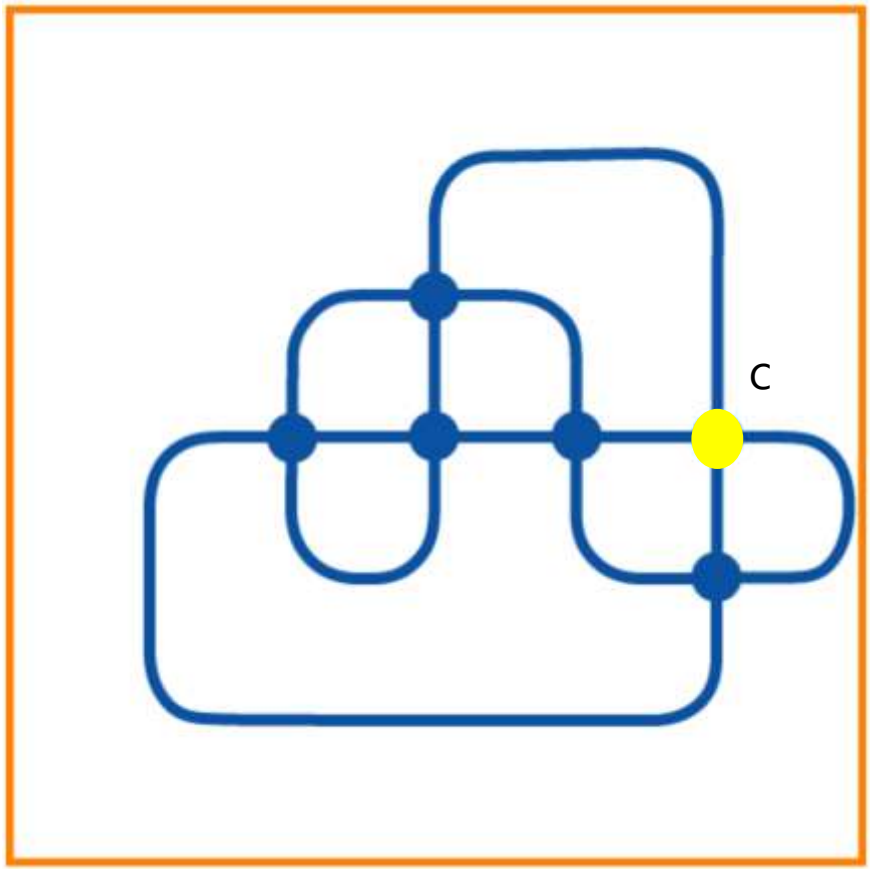


Checkerboard coloring

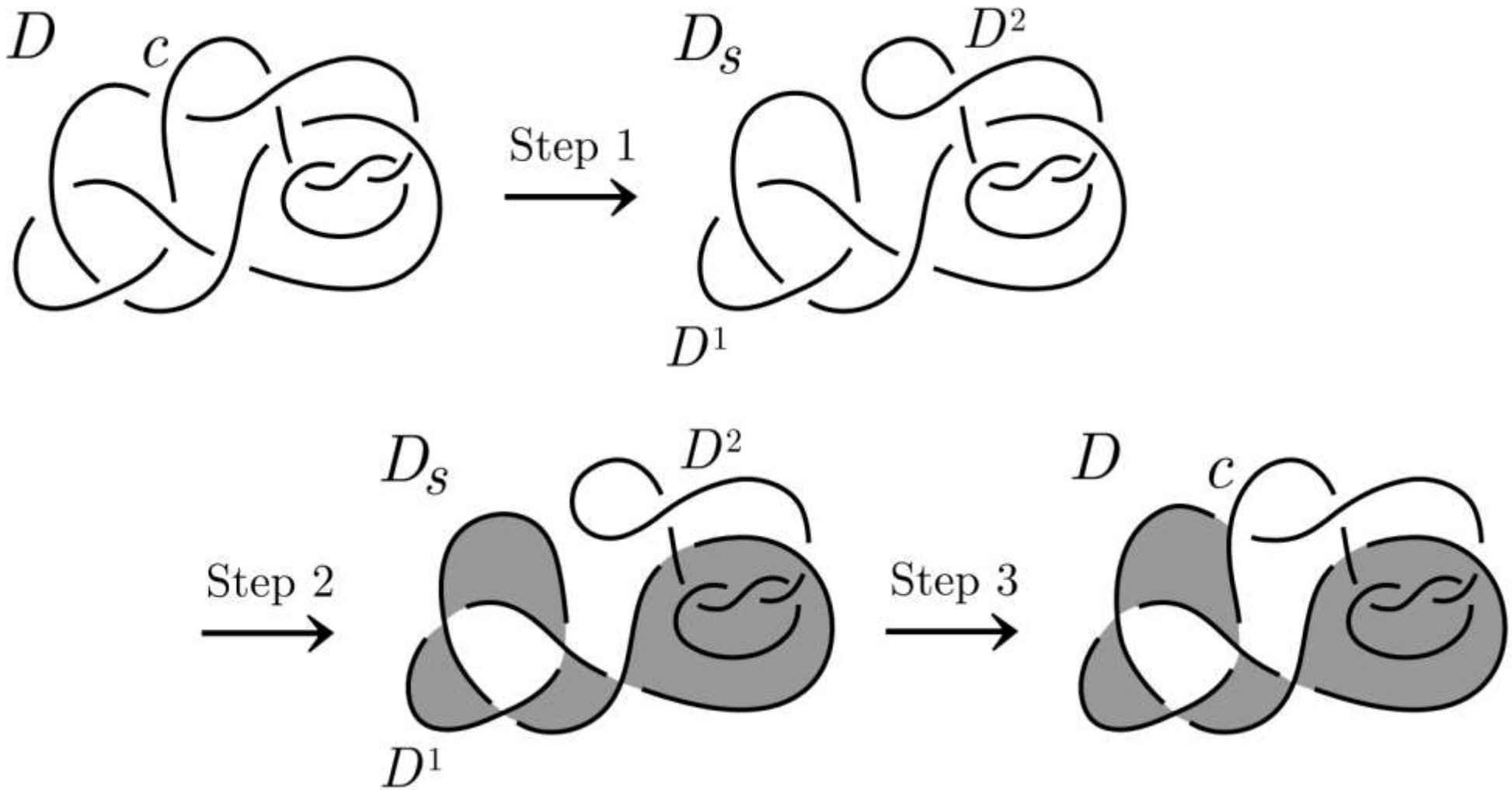
3. How to prove?

Step 5. Change crossing in shaded regions.

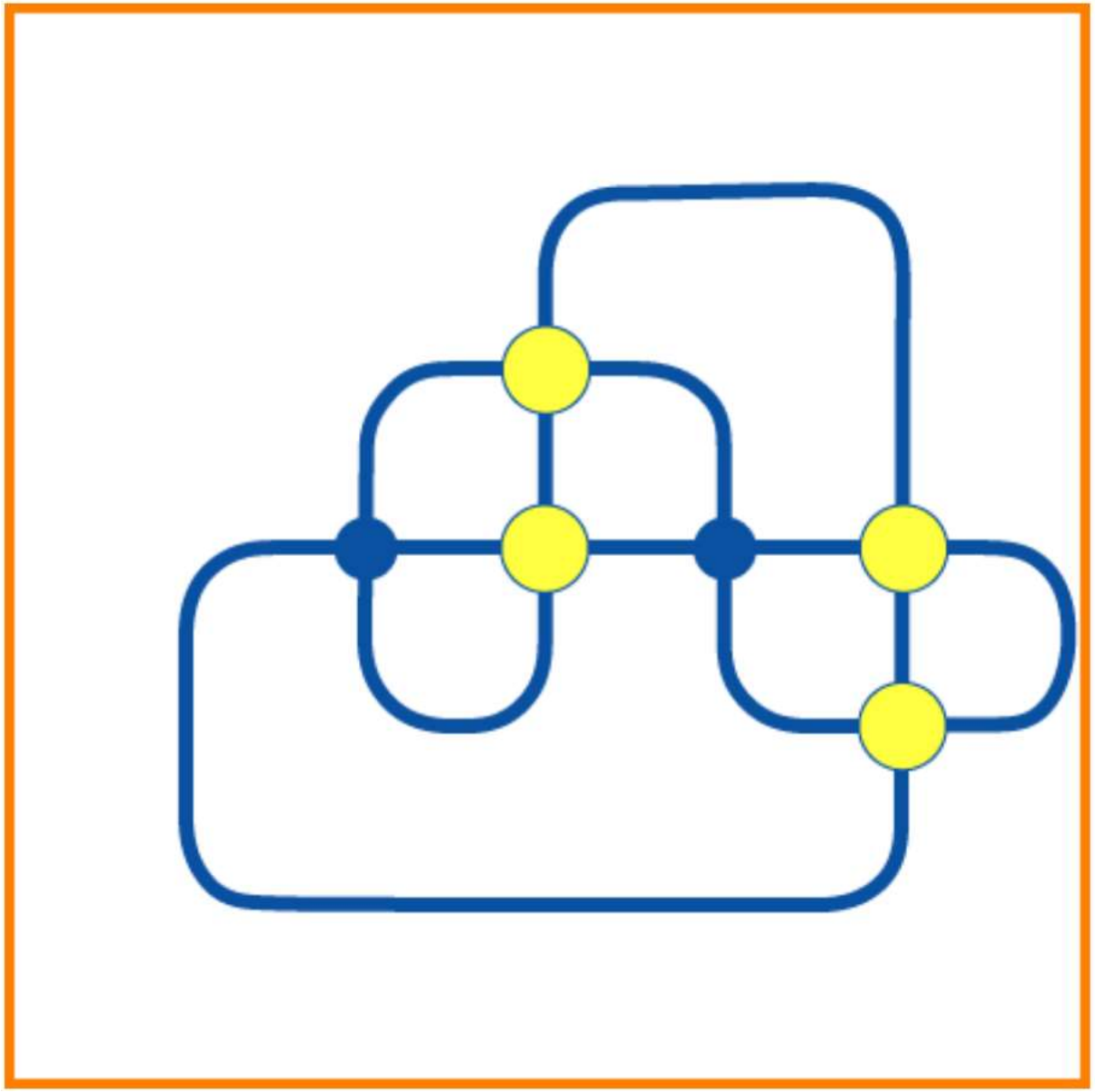


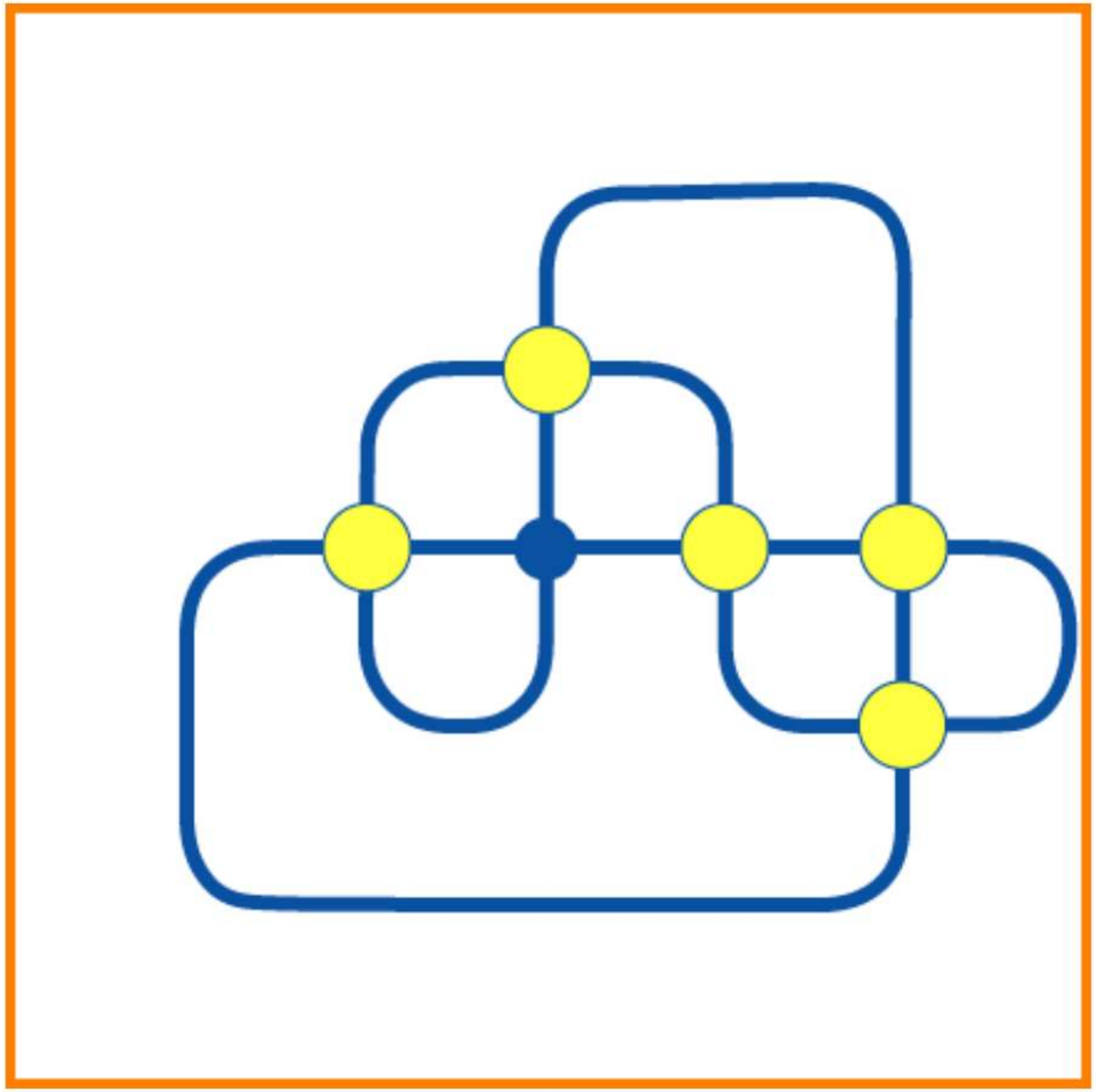


3. How to prove?

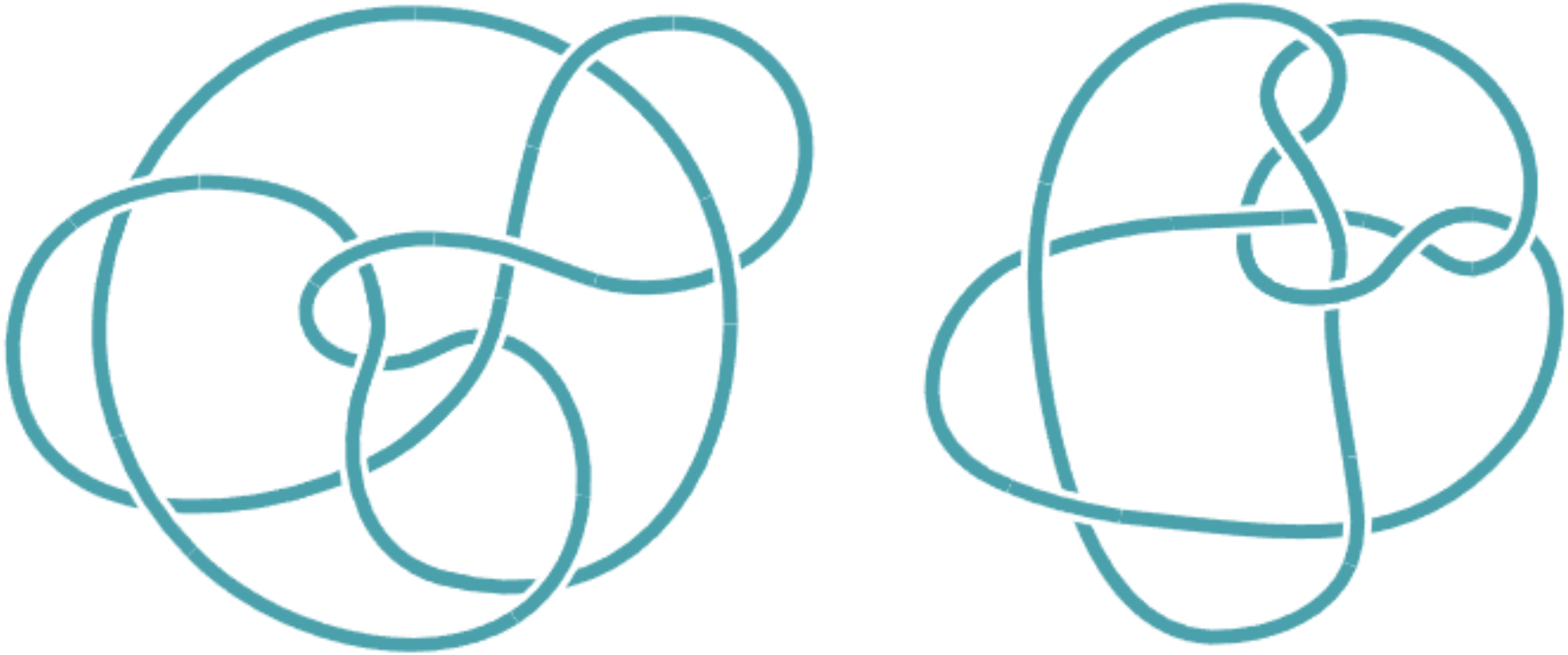


4. Use Algorithm!





5. Knot Theory



Are they same?

5. Knot Theory

https://knotinfo.math.indiana.edu/diagram_display.php?10_9

Source

- https://www.sci.osaka-cu.ac.jp/math/OCAMI/news/gamehp/etop/gametop.html
- Ayaka SHIMIZU. "Region crossing change is an unknotting operation. " J. Math. Soc. Japan 66 (3) 693 - 708, July, 2014. https://doi.org/10.2969/jmsj/06630693