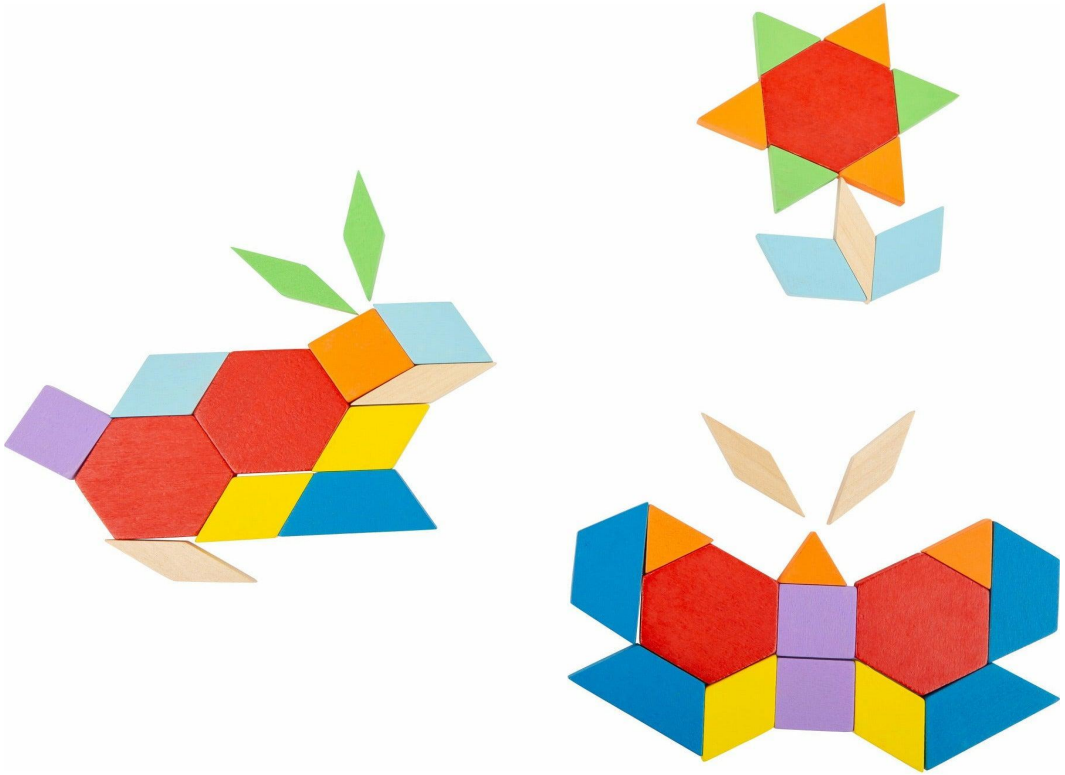
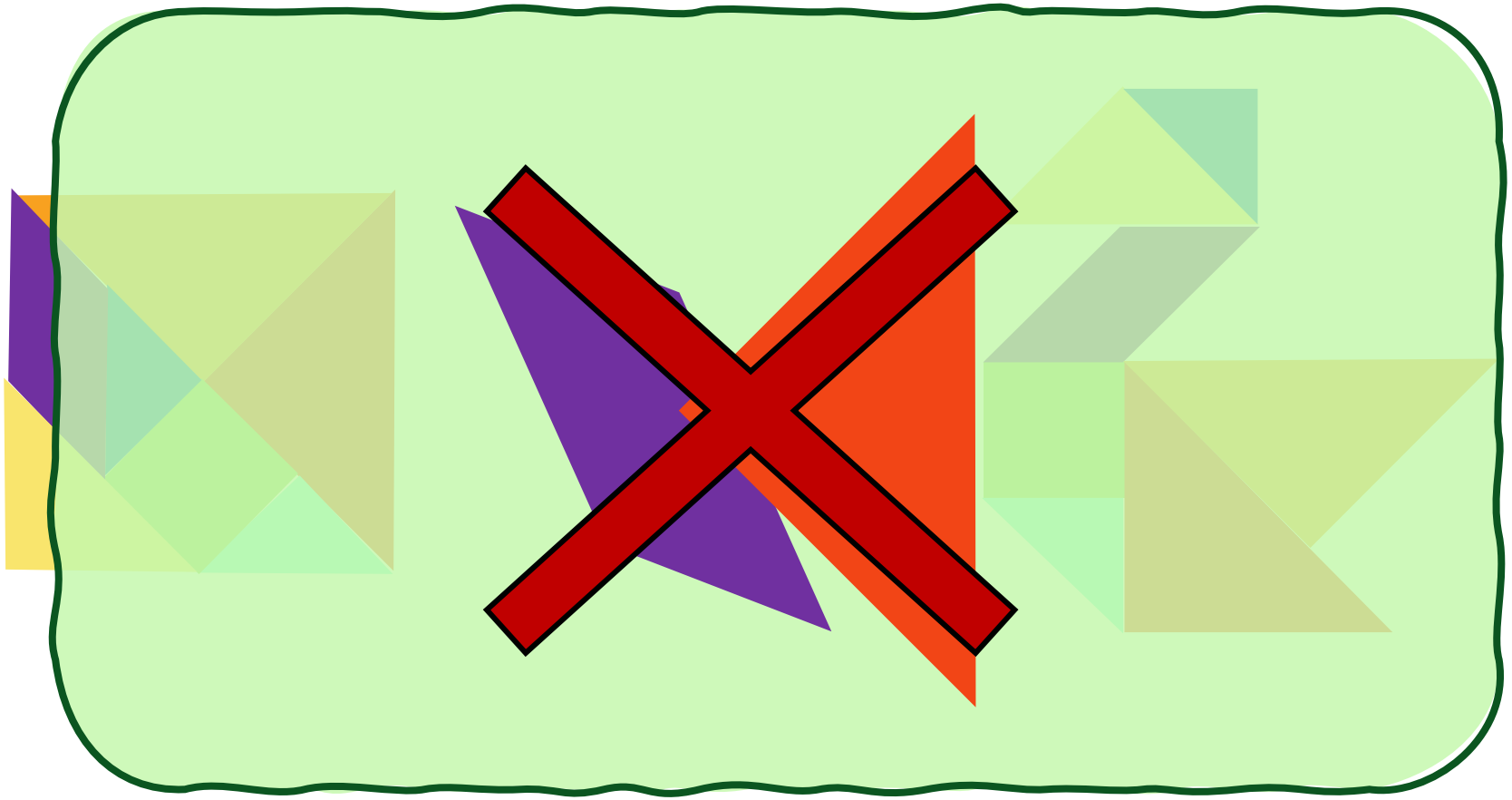
Scissors Congruence



Scissors Congruence in the plane



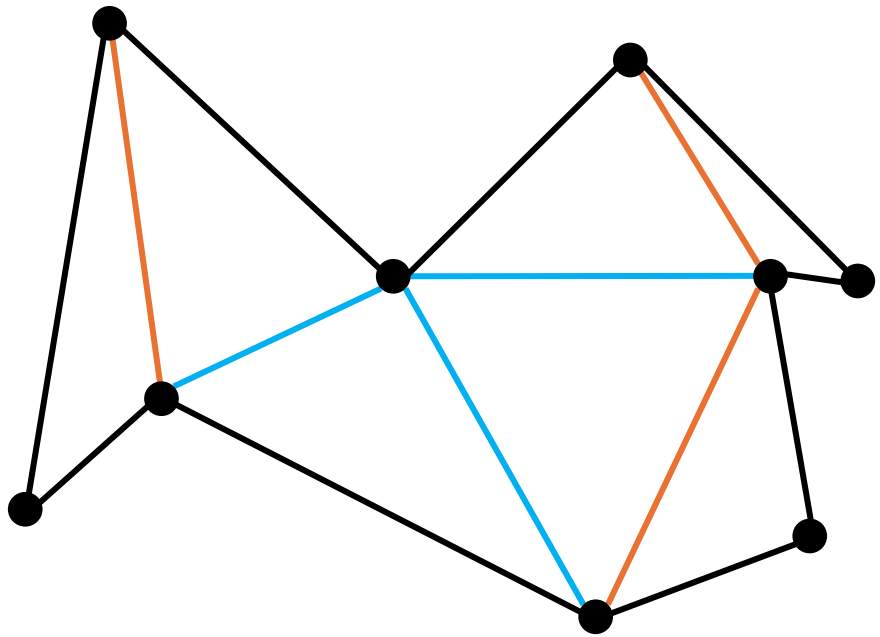
정리

2차원에서 다음은 동치이다

1) 두 다각형의 '넓이가 같다'

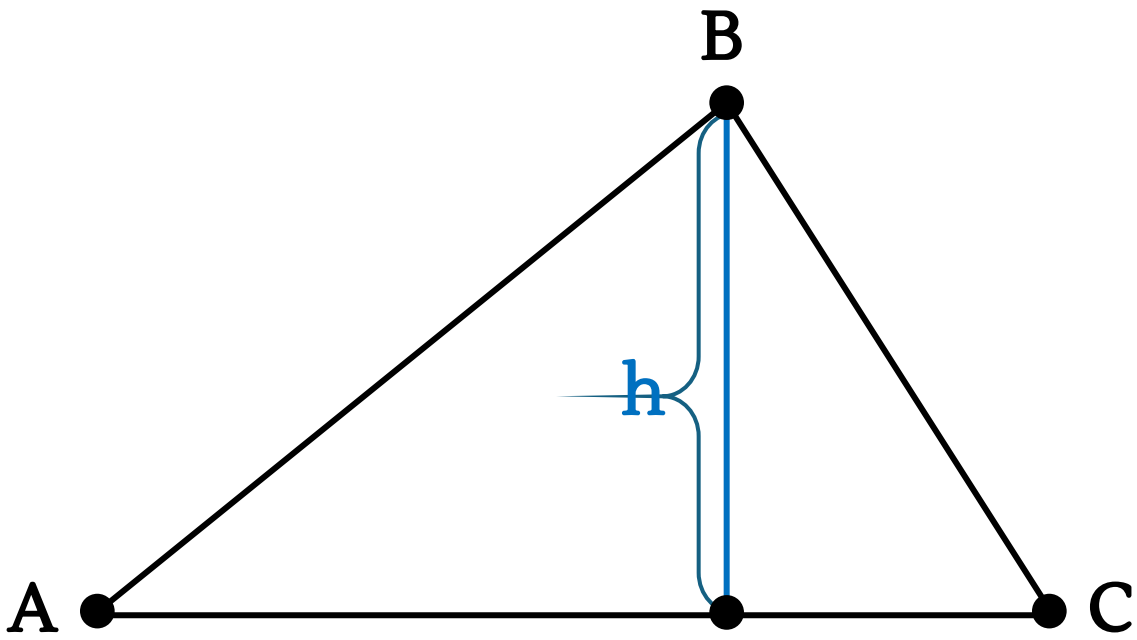
2) '가위 합동(Scissors Congruent)'이다 .

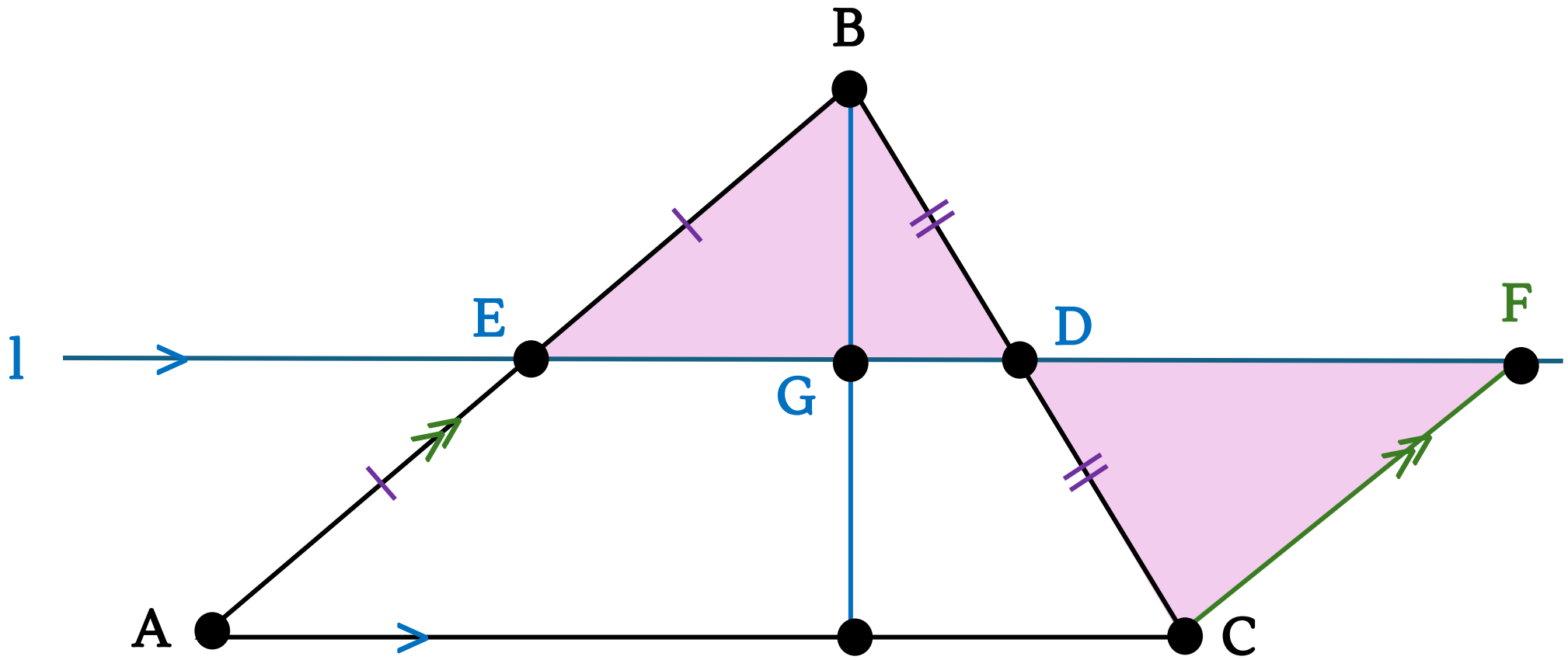
증명)

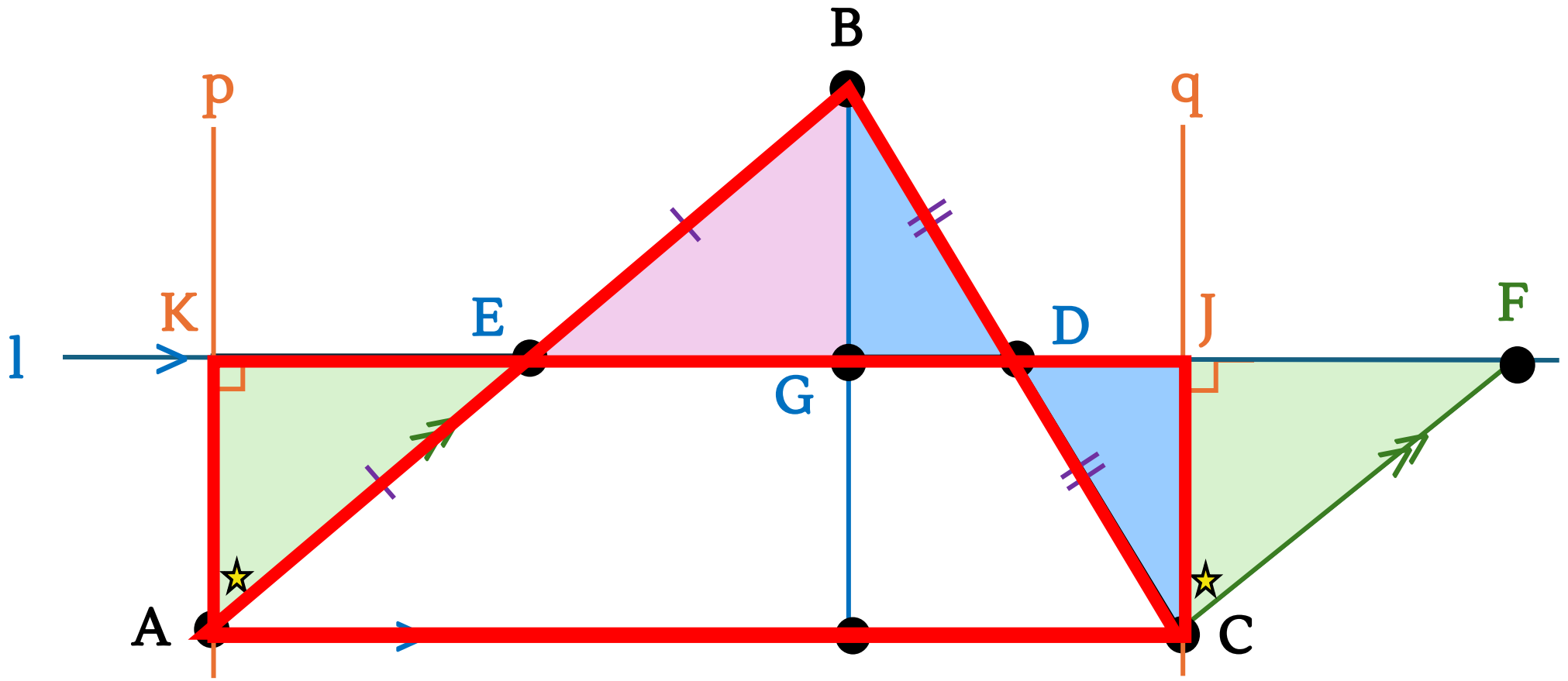


보조정리

임의의 삼각형은 밑변의 길이가 같은 직사각형과 가위 합동이다.

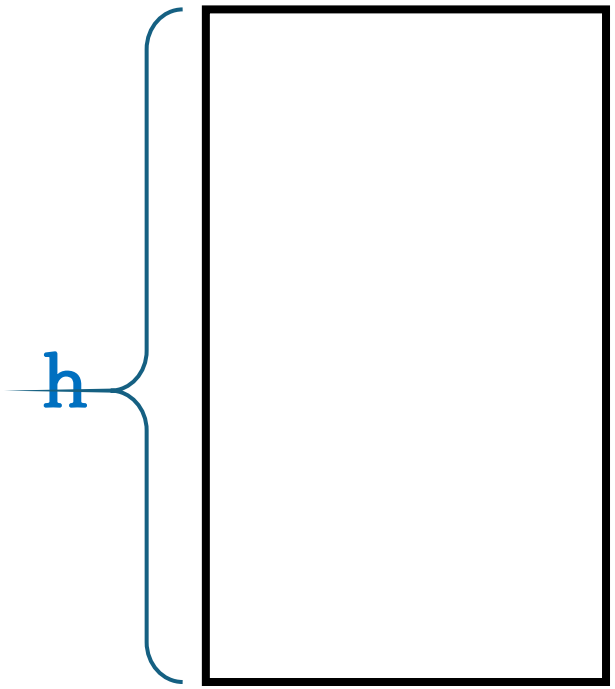



B
E
D
F
l
G
A
C

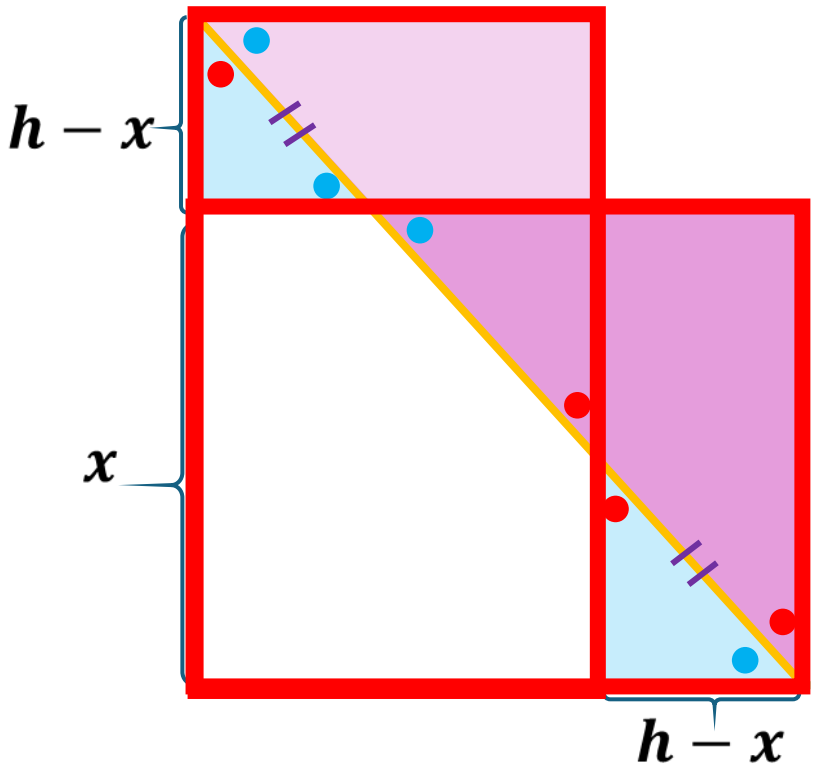

B
p
q
K
E
D
J
F
l
G
A
C

보조정리

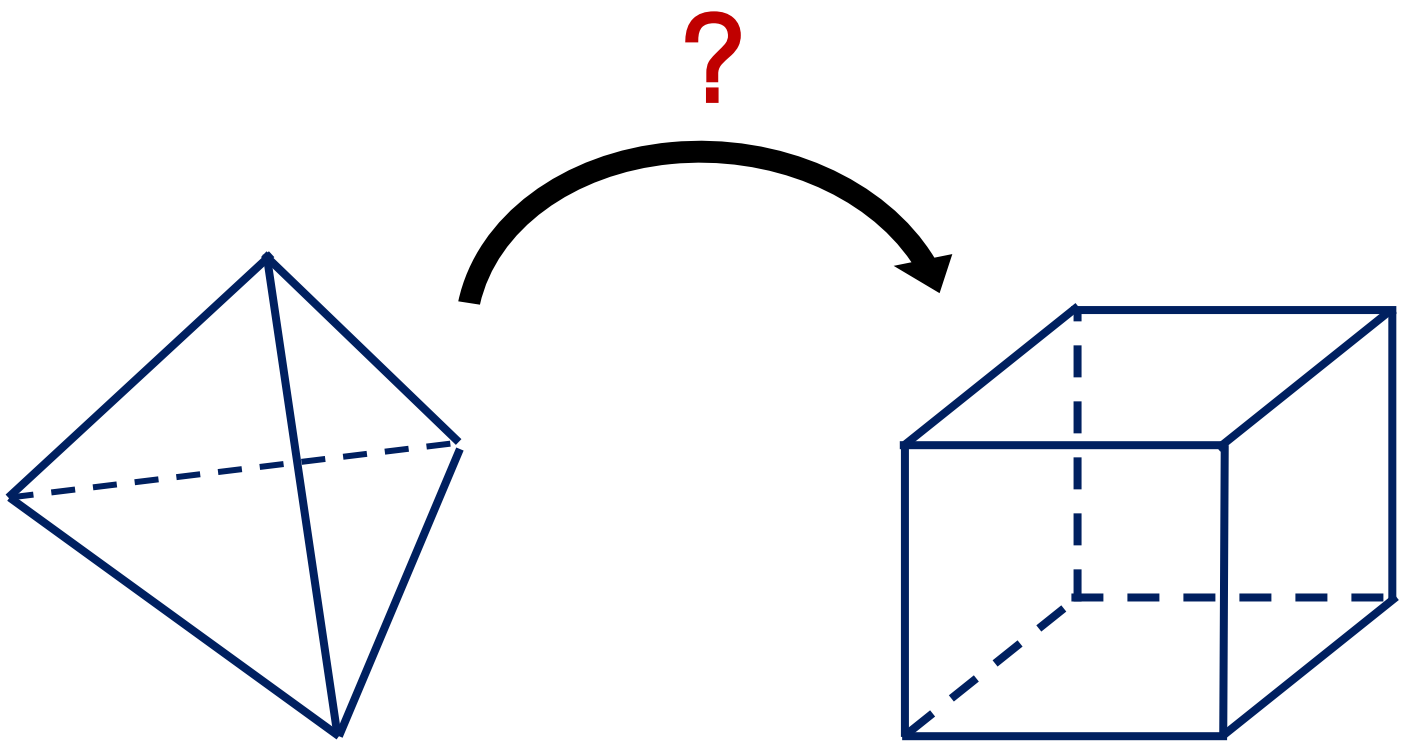
높이가 h 인 직사각형은 임의의 높이 x $(x < h)$를 가진 다른 직사각형과 가위합동이다.

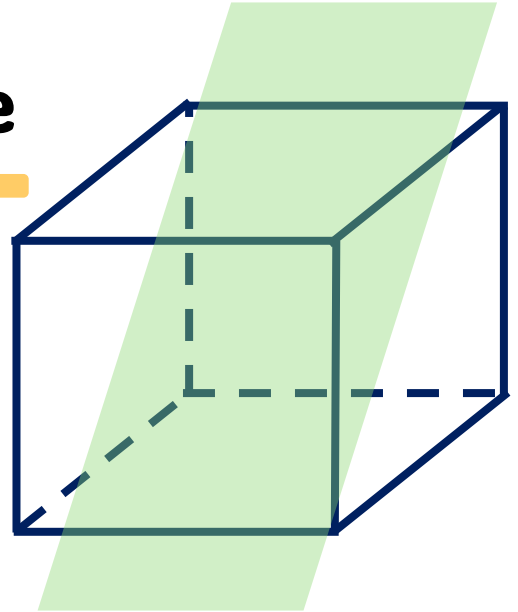


if $h - x < x$



Scissors Congruence in the 3-Space

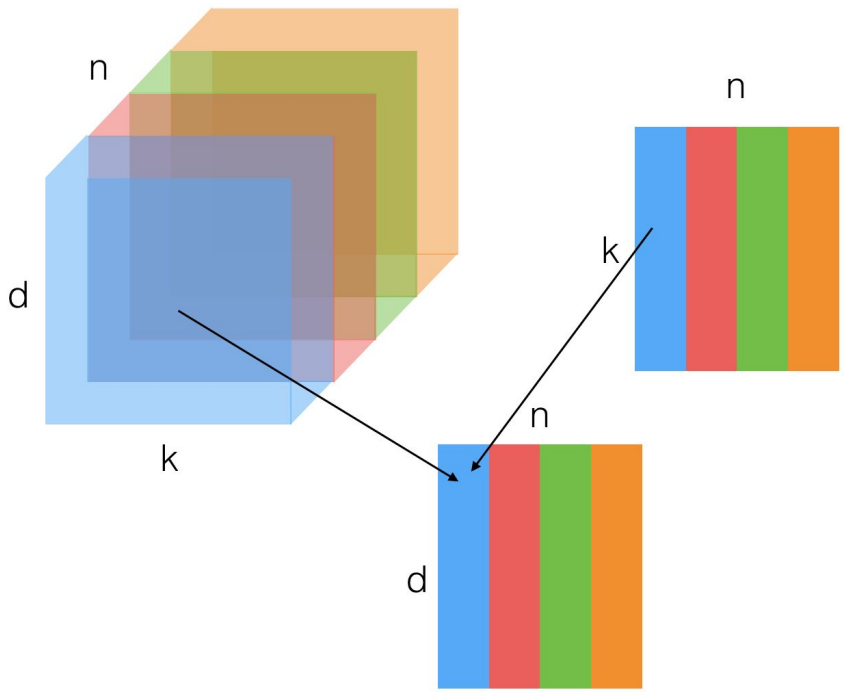


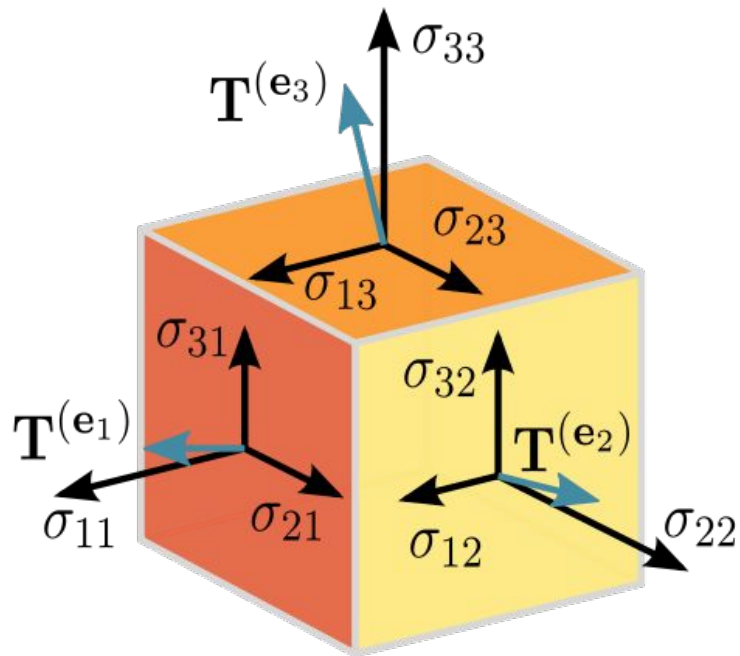


정리

가위 합동인 두 다면체의 부피는 같다.

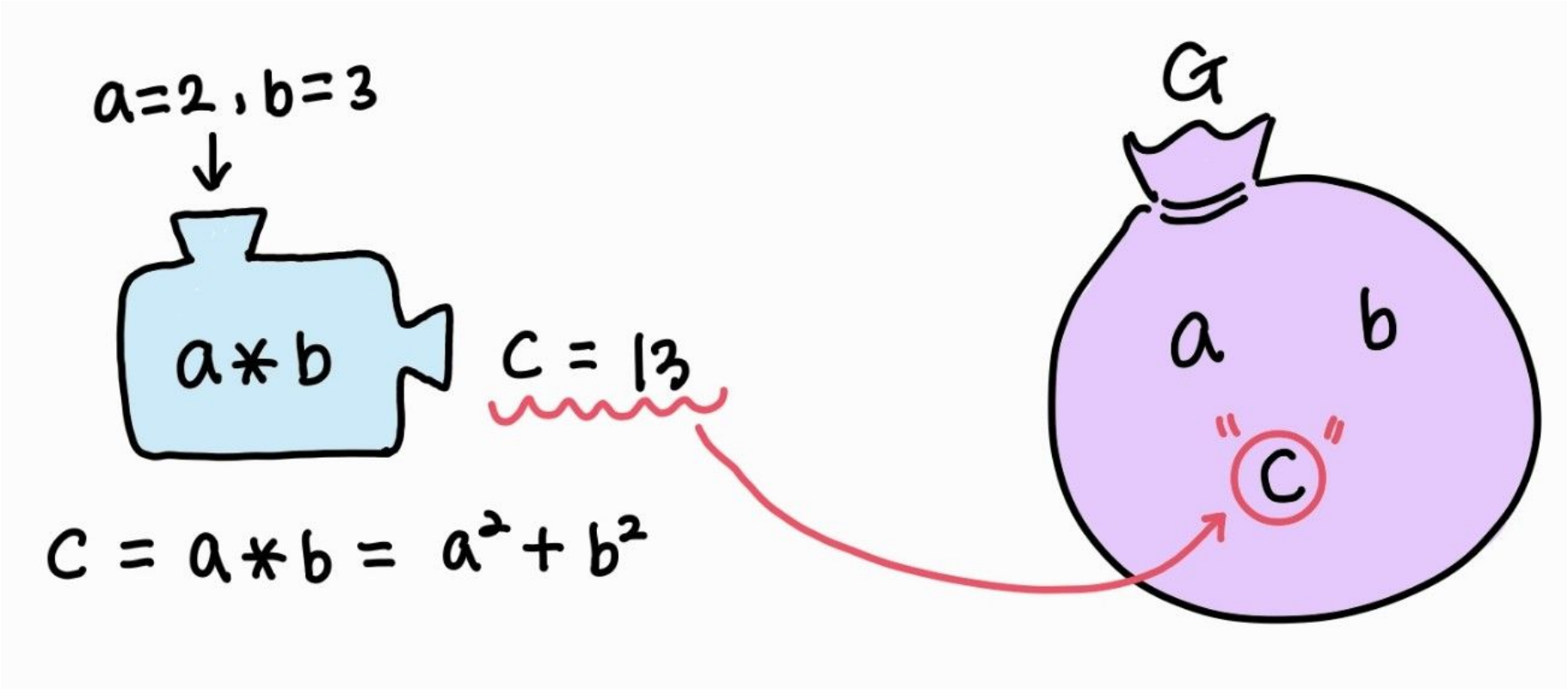
Tensor ⊗


n
d
k
n
k
n
d


σ_{33}
$\mathbf{T}^{(\mathbf{e}_3)}$
σ_{23}
σ_{13}
σ_{31}
σ_{32}
$\mathbf{T}^{(\mathbf{e}_1)}$
$\mathbf{T}^{(\mathbf{e}_2)}$
σ_{11}
σ_{21}
σ_{12}
σ_{22}

Group(군)

1) 군 $< G, * >$ 는 연산 $*$에 대해 닫혀있는 집합



2) 결합법칙(associativity) 만족

$$(a * b) * c = a * (b * c)$$

ex) (2+3)+5 = 2+(3+5)

3) 항등원(identity element) 존재

$$e * x = x * e = x$$

ex) identity in $< \mathbb{Z}, + >$ is 0

5+0 = 0+5 = 5

4) 역원(inverse element) 존재

$$a * a' = a' * a = e$$

ex) $< \mathbb{Z}, + >$

5+(-5) = (-5)+5 = 0

Example

1) $< \mathbb{Z}, + >$, $< \mathbb{R}, + >$, $< \mathbb{R} - \{0\}, \cdot >$

항등원: 1 역원: $\frac{1}{a}$

항등원: 0
역원: $-a$

2) $< \mathbb{Z}_2, + >$ → 2로 나눈 나머지

$\mathbb{Z}_2 = \{0, 1\}$ □ 짝수: 0 / 홀수: 1

항등원: 0

역원:
1 □ 1 , 0 □ 0

Q. $< \mathbb{Z}, \cdot >$ 는 group일까?

NO! → 2의역원 $\notin \frac{1}{2} \in \mathbb{Z}$

group 정의 확인!

1. 연산에 의해 닫혀 있는가
2. 결합법칙이 성립 하는가
3. 항등원이 있는가
4. 역원이 있는가

Abelian Group

$$a * b = b * a$$

ex) 5+3 = 3+5

Dehn Invariant

$$D(P) = \sum length(e) \otimes [\theta(e)] \in V$$

P: polyhedron (다면체)

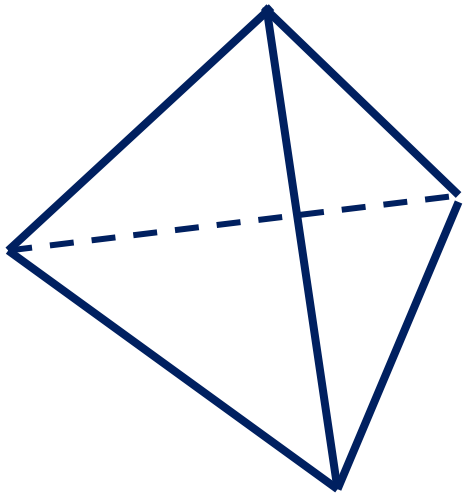
$V = \mathbb{R} \otimes \mathbb{R}/\pi\mathbb{Q}$

$\theta(e)$: 선분 e에서의 이면각

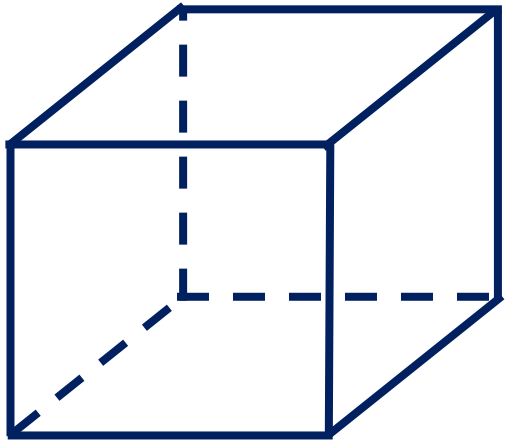
중요!! : 모든 선분에 대해 더해준다.

정리

두 다면체가 가위 합동이라면 부피와 Dehn invariant가 같다.



T



C

$$D(T) \neq D(C)$$

∴ 부피가 1인 정육면체 C 와 정사면체 T는 가위 합동일 수 없다.