바보들의 수학

수학과 23학번 이민혁

$$\frac{a}{b} + \frac{c}{d} = ?$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$ ✗

$$\frac{a}{b} + \frac{c}{d} = \frac{a + c}{b + d}$$ ✓

더하기를 새롭게 정의

$$\Downarrow$$

$$\frac{a}{b} \oplus \frac{c}{d} := ?$$

$$\mathbb{Q} = \left\{ \frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0 \right\}$$

$$\Downarrow$$

$$\frac{0}{1}, \frac{1}{3}, \frac{2}{4}$$

유리수 중에서 기약분수 끼리
덧셈만 다룸

$$\frac{2}{4} \oplus \frac{1}{3}$$ ✗

$$\frac{1}{2} \oplus \frac{1}{3}$$ ✓

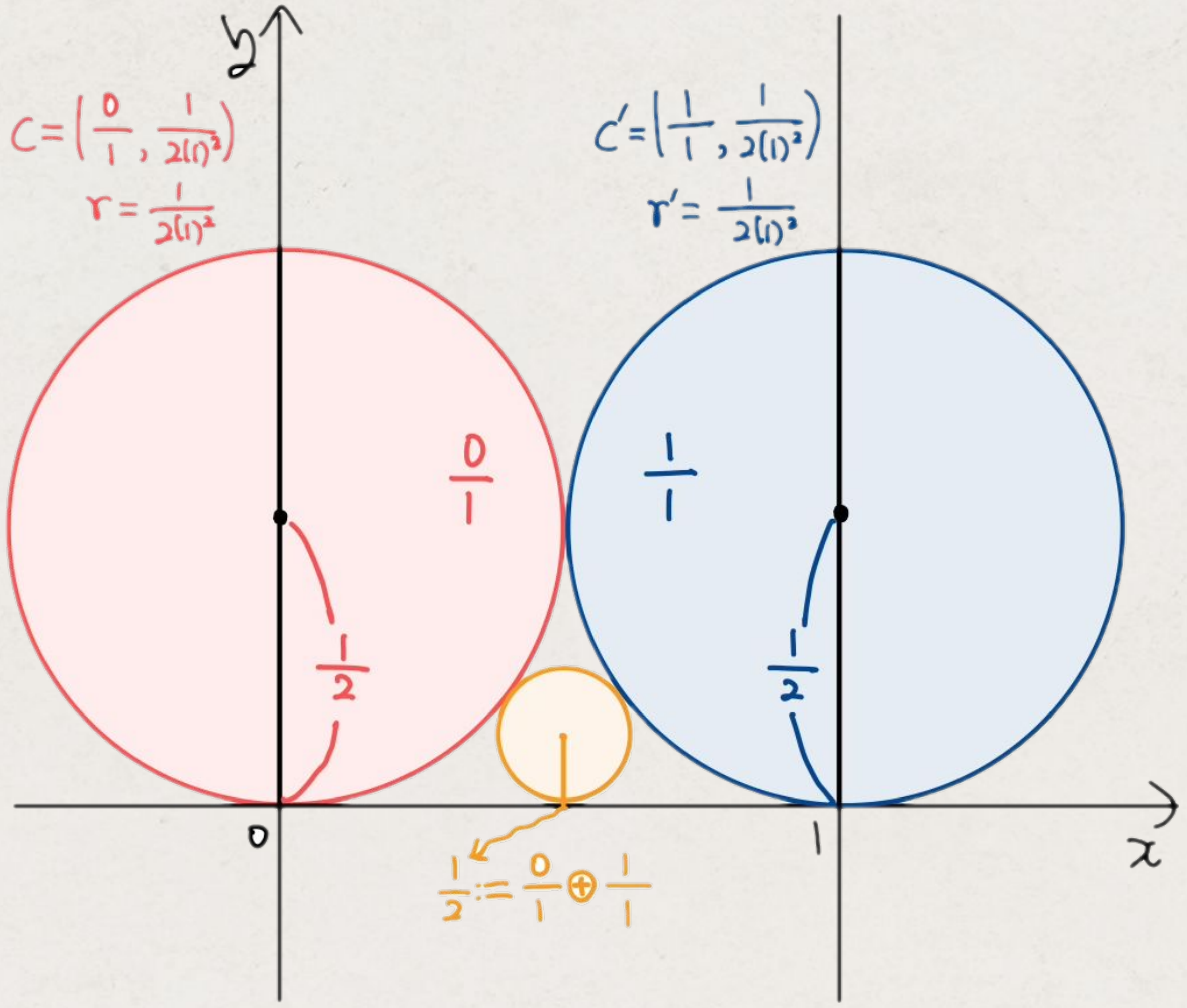
$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a + c}{b + d}$$

$$\frac{a}{b} \oplus \frac{c}{d} :=$$

반지름 r을 $\frac{1}{2b^2}, r' = \frac{1}{2d^2}$로 하고
중심을 $\left(\frac{a}{b}, r\right), \left(\frac{c}{d}, r'\right)$으로 하는
두 원과 x축과 접하는
원의 중심의 x성분

$$ex)\ \frac{0}{1} \oplus \frac{1}{1}$$



$ex)\ \frac{0}{1} \oplus \frac{1}{1}$

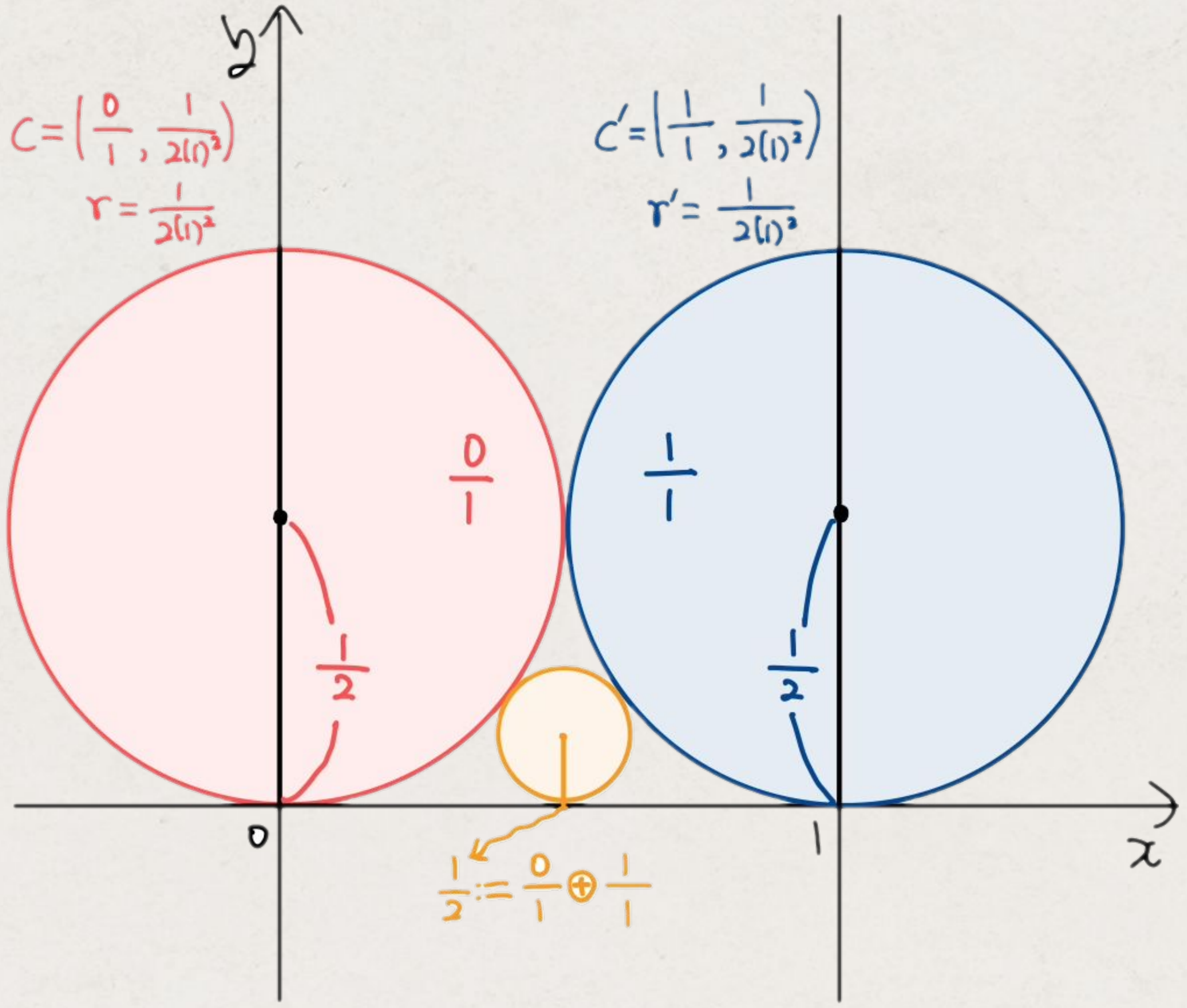
$$\frac{0}{1} \oplus \frac{1}{1} = \frac{1}{2} = \frac{0+1}{1+1}$$

$ex)\ \frac{1}{2} \oplus \frac{1}{1}$

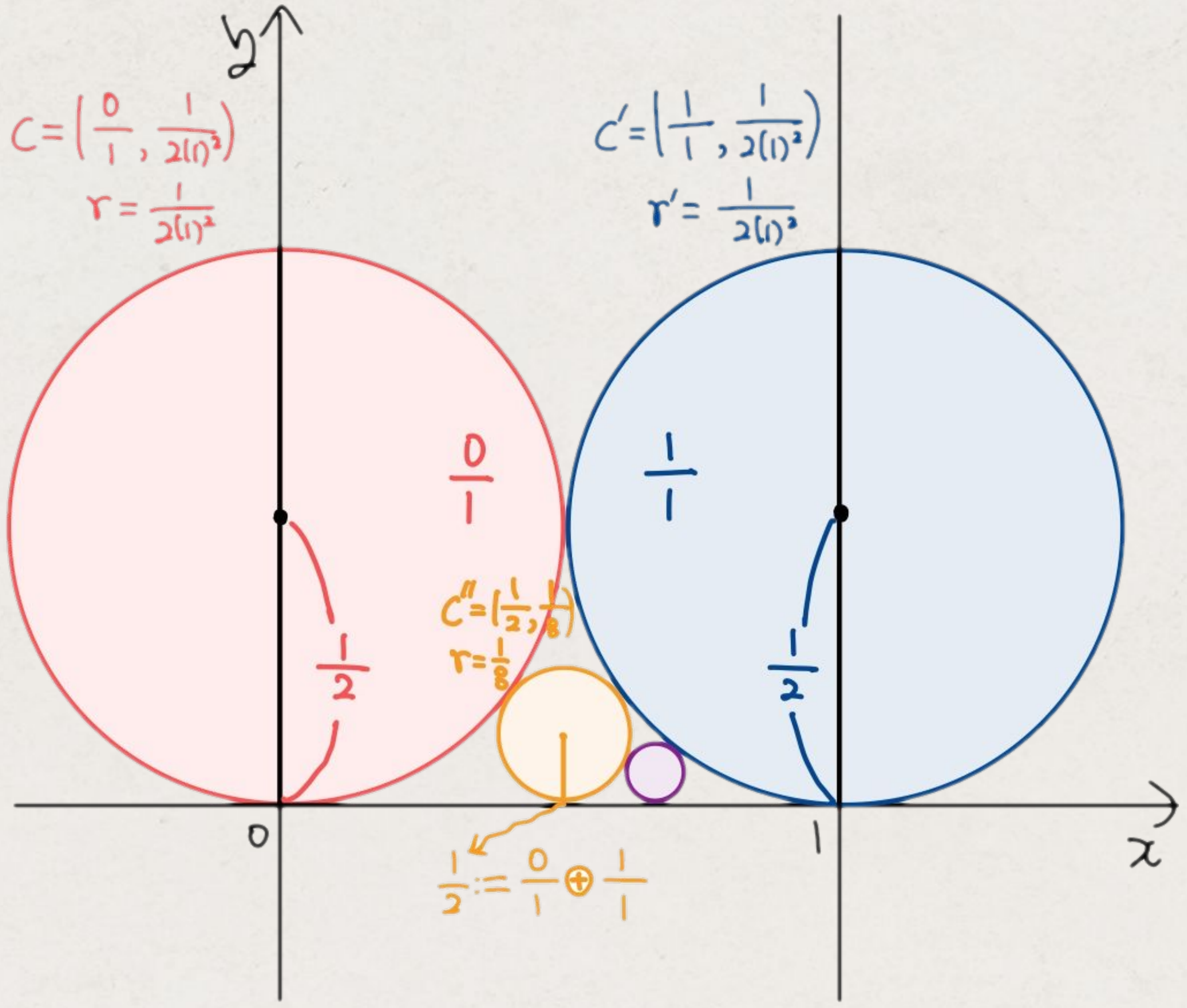
$$\frac{0}{1} \oplus \frac{1}{1} = \frac{1}{2} = \frac{0+1}{1+1}$$

$$\frac{1}{2} \oplus \frac{1}{1} = \frac{1+1}{2+1} = \frac{2}{3} \ldots ?$$

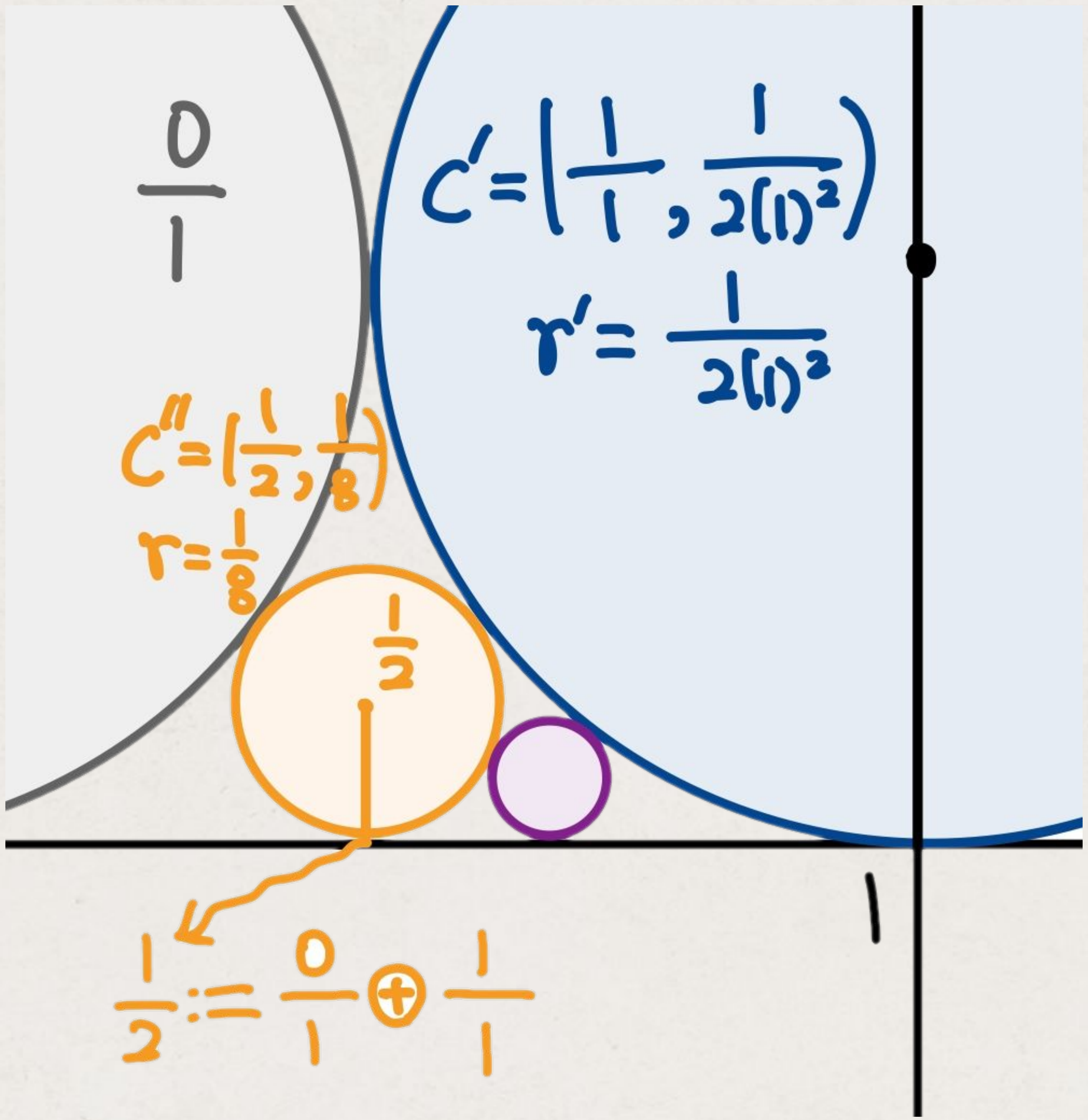
$$ex)\ \frac{1}{2} \oplus \frac{1}{1}$$



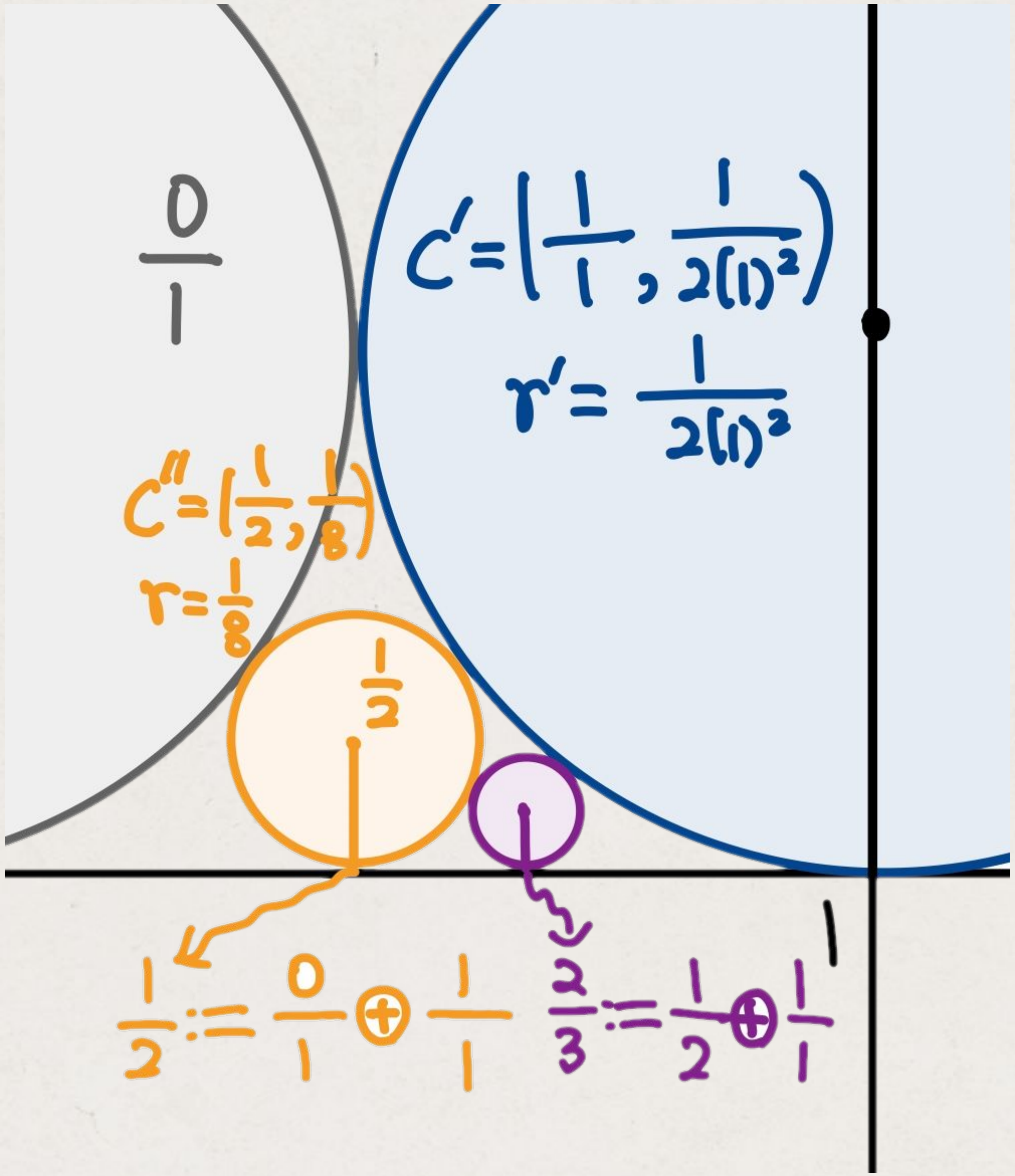
$ex)\ \frac{1}{2} \oplus \frac{1}{1}$



$ex)\ \frac{1}{2} \oplus \frac{1}{1}$



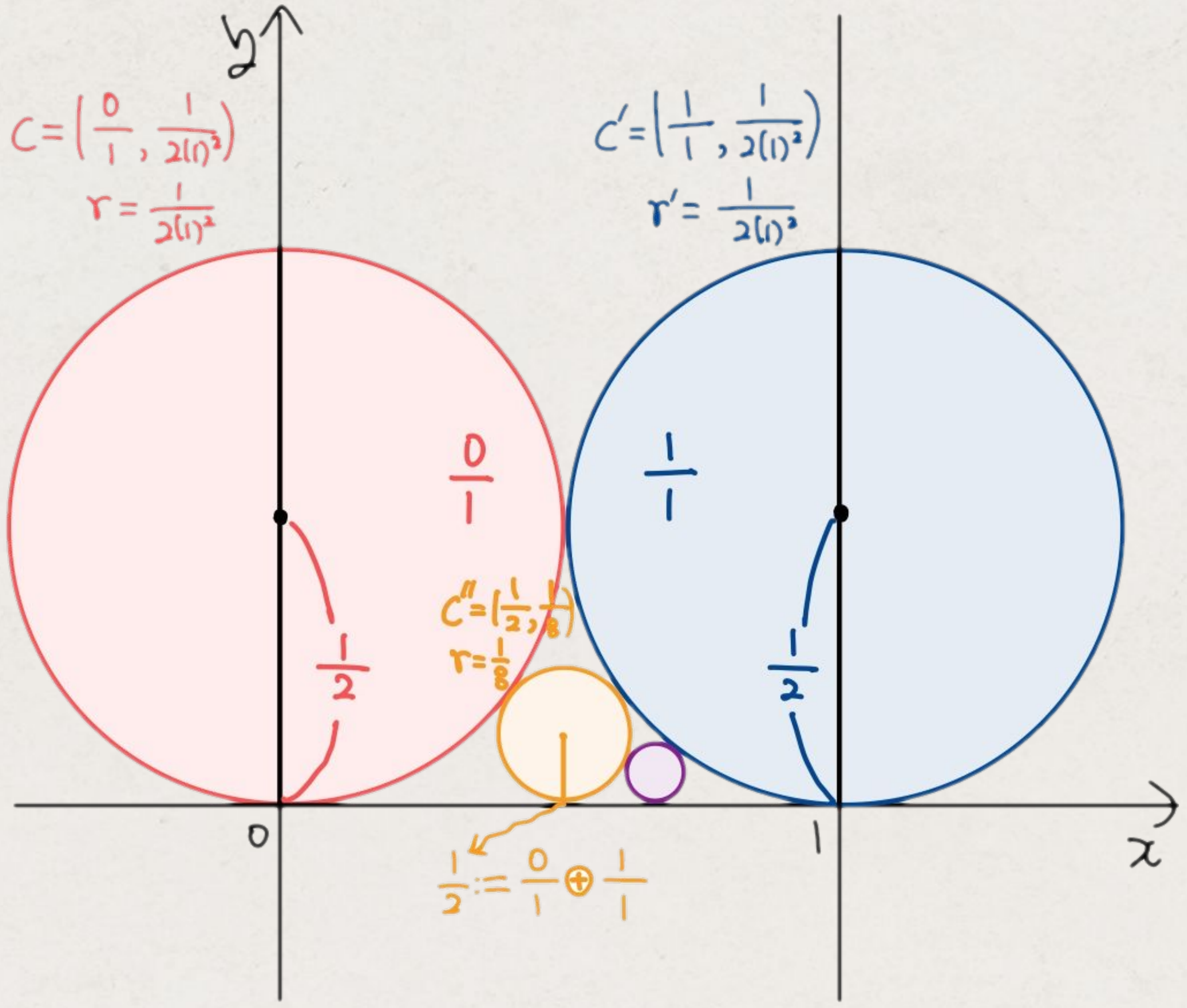
$ex)\ \frac{1}{2} \oplus \frac{1}{1}$

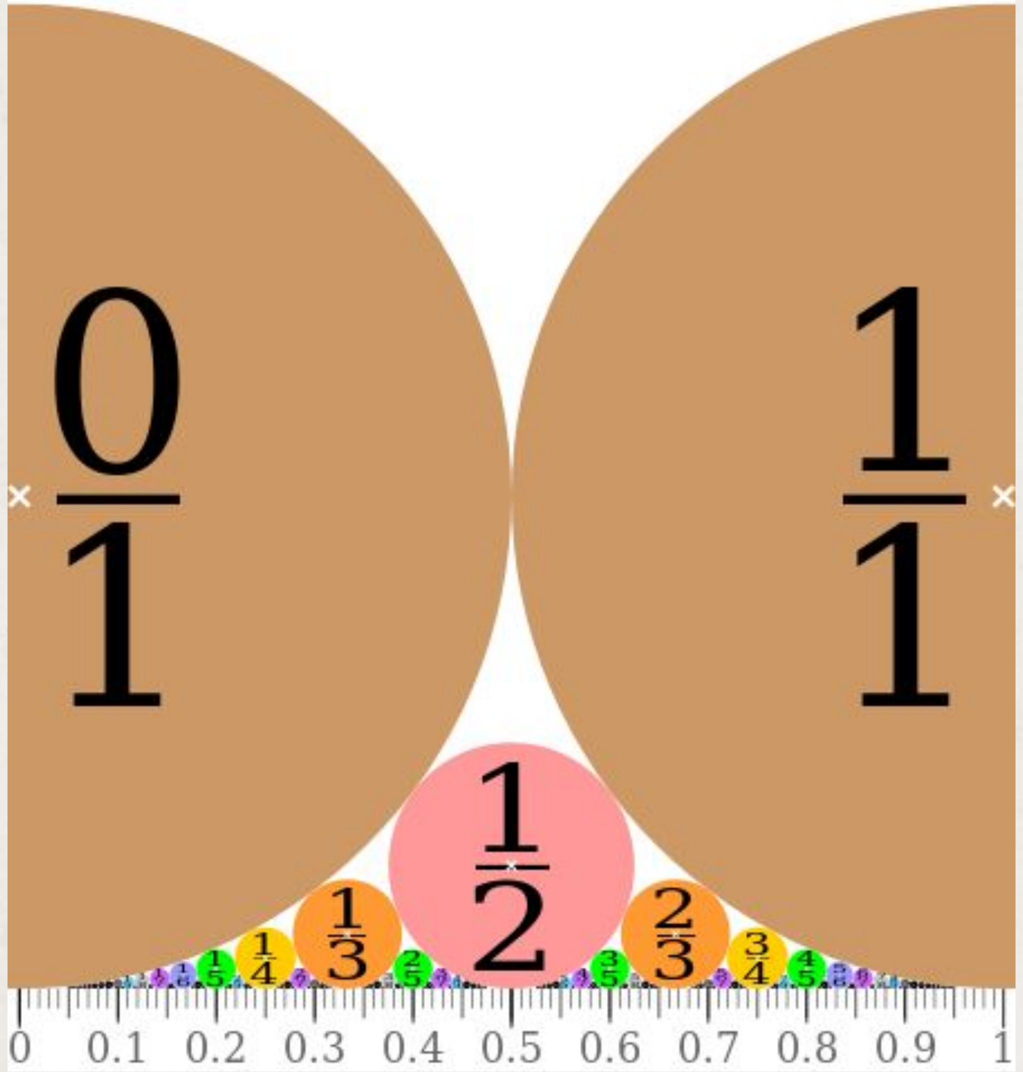


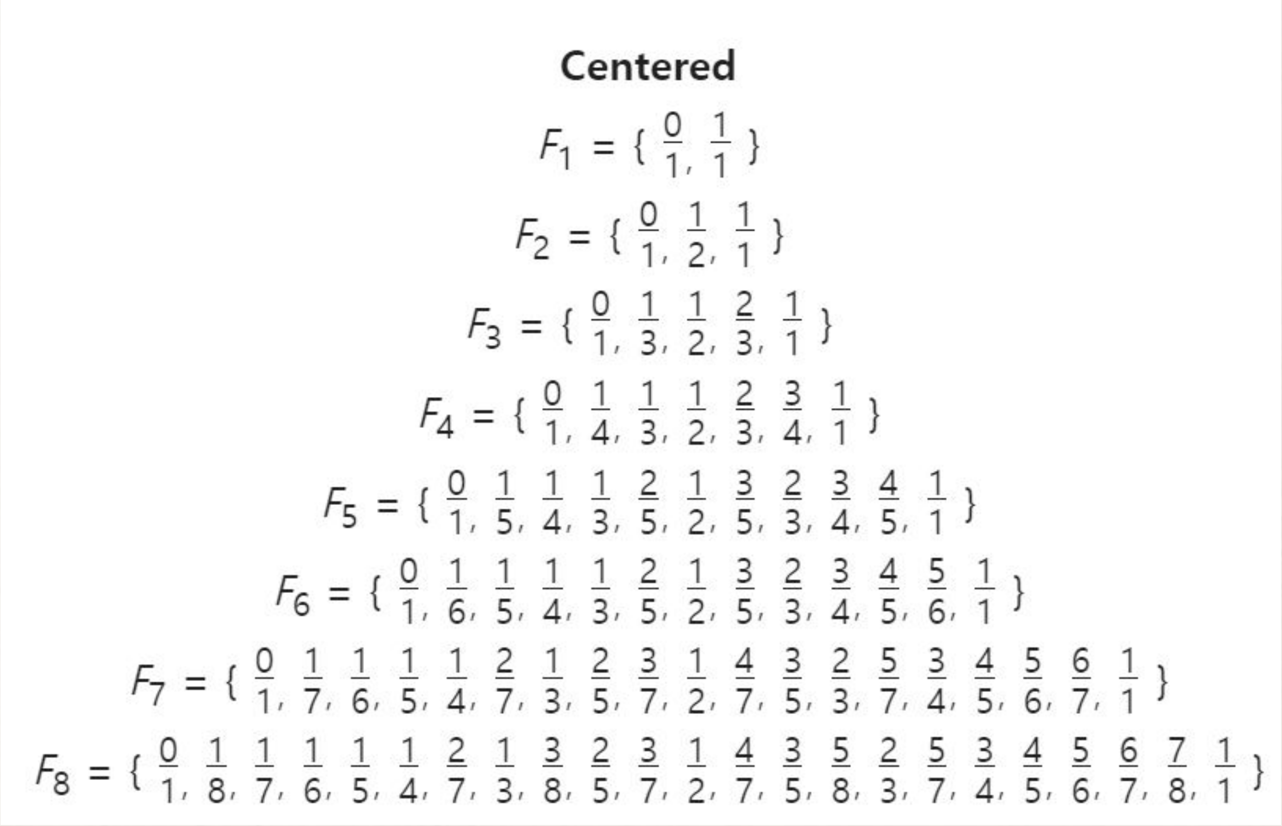
$ex)\ \frac{1}{2} \oplus \frac{1}{1}$

$$\frac{1}{2} \oplus \frac{1}{1} = \frac{2}{3} = \frac{1+1}{2+1}$$

두 유리수를 항상 더할 수 있냐?


y
$C=\left(\frac{0}{1}, \frac{1}{2(1)^2}\right)$
$r=\frac{1}{2(1)^2}$
$C'=\left(\frac{1}{1}, \frac{1}{2(1)^2}\right)$
$r'=\frac{1}{2(1)^2}$
$\frac{0}{1}$
$\frac{1}{1}$
$\frac{1}{2}$
$C''=\left(\frac{1}{2}, \frac{1}{8}\right)$
$r=\frac{1}{8}$
$\frac{1}{2}$
0
1
x
$\frac{1}{2} := \frac{0}{1} \oplus \frac{1}{1}$





Centered

$$F_1 = \{ \tfrac{0}{1}, \tfrac{1}{1} \}$$

$$F_2 = \{ \tfrac{0}{1}, \tfrac{1}{2}, \tfrac{1}{1} \}$$

$$F_3 = \{ \tfrac{0}{1}, \tfrac{1}{3}, \tfrac{1}{2}, \tfrac{2}{3}, \tfrac{1}{1} \}$$

$$F_4 = \{ \tfrac{0}{1}, \tfrac{1}{4}, \tfrac{1}{3}, \tfrac{1}{2}, \tfrac{2}{3}, \tfrac{3}{4}, \tfrac{1}{1} \}$$

$$F_5 = \{ \tfrac{0}{1}, \tfrac{1}{5}, \tfrac{1}{4}, \tfrac{1}{3}, \tfrac{2}{5}, \tfrac{1}{2}, \tfrac{3}{5}, \tfrac{2}{3}, \tfrac{3}{4}, \tfrac{4}{5}, \tfrac{1}{1} \}$$

$$F_6 = \{ \tfrac{0}{1}, \tfrac{1}{6}, \tfrac{1}{5}, \tfrac{1}{4}, \tfrac{1}{3}, \tfrac{2}{5}, \tfrac{1}{2}, \tfrac{3}{5}, \tfrac{2}{3}, \tfrac{3}{4}, \tfrac{4}{5}, \tfrac{5}{6}, \tfrac{1}{1} \}$$

$$F_7 = \{ \tfrac{0}{1}, \tfrac{1}{7}, \tfrac{1}{6}, \tfrac{1}{5}, \tfrac{1}{4}, \tfrac{2}{7}, \tfrac{1}{3}, \tfrac{2}{5}, \tfrac{3}{7}, \tfrac{1}{2}, \tfrac{4}{7}, \tfrac{3}{5}, \tfrac{2}{3}, \tfrac{5}{7}, \tfrac{3}{4}, \tfrac{4}{5}, \tfrac{5}{6}, \tfrac{6}{7}, \tfrac{1}{1} \}$$

$$F_8 = \{ \tfrac{0}{1}, \tfrac{1}{8}, \tfrac{1}{7}, \tfrac{1}{6}, \tfrac{1}{5}, \tfrac{1}{4}, \tfrac{2}{7}, \tfrac{1}{3}, \tfrac{3}{8}, \tfrac{2}{5}, \tfrac{3}{7}, \tfrac{1}{2}, \tfrac{4}{7}, \tfrac{3}{5}, \tfrac{5}{8}, \tfrac{2}{3}, \tfrac{5}{7}, \tfrac{3}{4}, \tfrac{4}{5}, \tfrac{5}{6}, \tfrac{6}{7}, \tfrac{7}{8}, \tfrac{1}{1} \}$$

Farey sequence

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots + \cfrac{1}{a_n}}}} := [a_0; a_1, a_2, \ldots, a_n]$$

Continued fraction(연분수)

$ex)\ \frac{0}{1}, \frac{1}{3}, \frac{1}{2}$

$$\frac{1}{3} = 0 + \cfrac{1}{2 + \cfrac{1}{1}} \coloneqq [0; 2, 1]$$

$$\frac{1}{2} = 0 + \frac{1}{2} \coloneqq [0; 2]$$

$$\frac{1}{3} = 0 + \cfrac{1}{2 + \cfrac{1}{1}} \coloneqq [0; 2, 1]$$

$$\frac{1}{2} = 0 + \frac{1}{2} \coloneqq [0; 2] \qquad \frac{0}{1} = 0 \coloneqq [0;]$$

Centered

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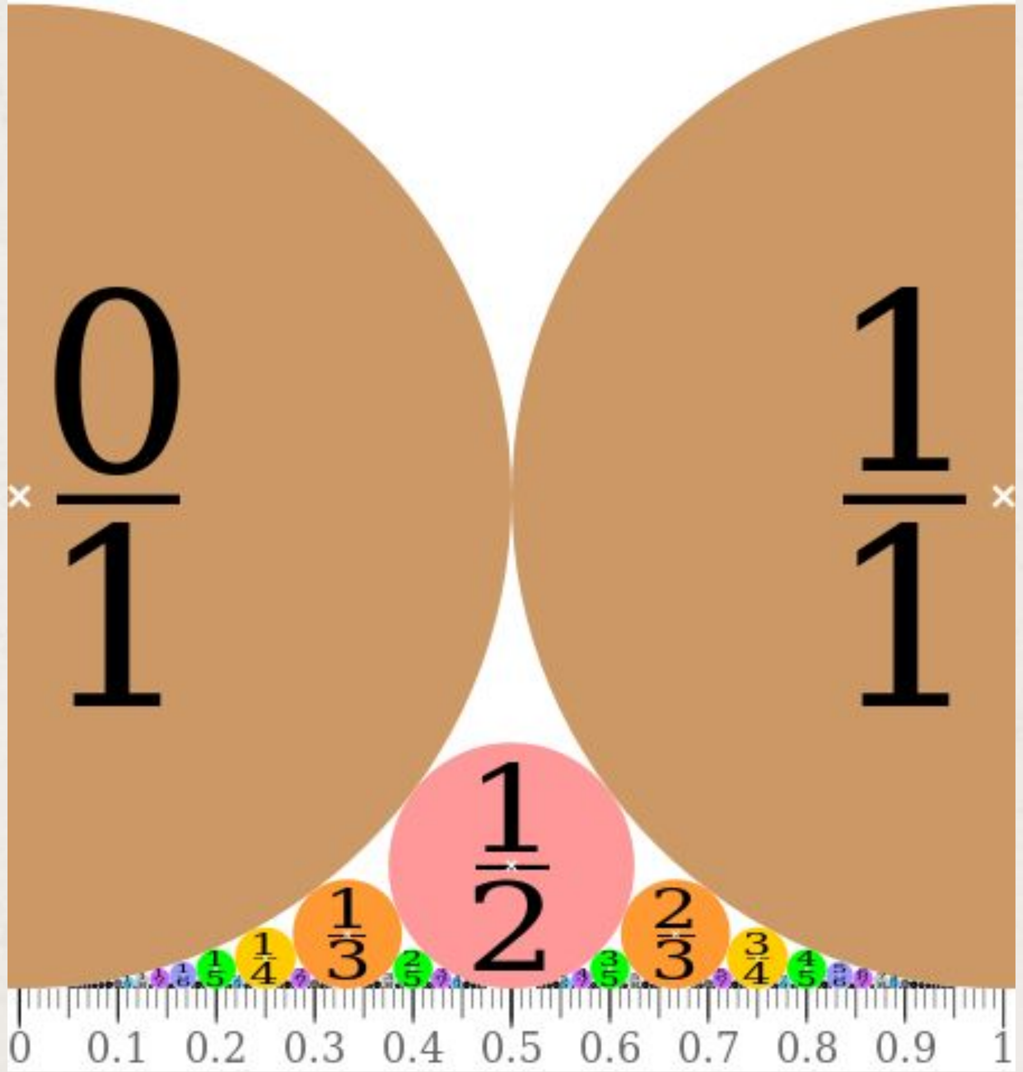
Farey sequence

$$[0; a_1, a_2, \ldots, a_{n-1}, a_n, 1]$$

$$\Downarrow$$

$$[0; a_1, a_2, \ldots, a_n]$$

$$[0; a_1, a_2, \ldots, a_{n-1}]$$



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Farey sequence

$ex)\ \frac{1}{3}, \frac{3}{8}, \frac{2}{5}$

$$\frac{3}{8} = 0 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}} := [0; 2, 1, 1, 1]$$

$$\Downarrow$$

$$[0; 2, 1, 1]$$

$$[0; 2, 1]$$

$$[0; 2,1,1] = 0 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1}}} = \frac{2}{5}$$

$$[0; 2,1,1] = 0 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1}}} = \frac{2}{5}$$

$$[0; 2,1] = 0 + \cfrac{1}{2 + \cfrac{1}{1}} = \frac{1}{3}$$

Centered

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Farey sequence

$$F_n := \{a_{k,n} : k = 0, 1, \ldots, m_n\}$$
$$d_{k,n} = a_{k,n} - k/m_n$$

$$\Downarrow$$

$$\sum_{k=1}^{m_n} d_{k,n}^2 = O(n^r)\ \forall r > -1$$

더하기를 새롭게 정의

$$\Downarrow$$

$$\frac{a}{b} \oplus \frac{c}{d} \coloneqq \frac{a+c}{b+d}$$