무한급수의 신비

$1-1+1/2-1/2+1/3-1/3\ldots\neq 0?$

목차

$$\mathbf{1 - 1 + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} \cdots}$$

$$1 - 1 + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} \cdots$$

$$\Downarrow$$

$$\underbrace{1 + \frac{1}{2}}_{\text{양수 2개}} \underbrace{- 1}_{\text{음수 1개}} \underbrace{+ \frac{1}{3} + \frac{1}{4}}_{\text{양수 2개}} \underbrace{- \frac{1}{2}}_{\text{음수 1개}} \cdots$$

$$1+\frac{1}{2}-1+\frac{1}{3}+\frac{1}{4}-\frac{1}{2}\cdots$$

$$=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\cdots$$

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\cdots$$

$$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)\cdots>0$$

?

$$\mathbf{1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\ldots=\boldsymbol{ln2}}$$

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\cdots=$$

$$1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}\cdots=$$

$$1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}\cdots=$$

$$\frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\frac{1}{10}\cdots=$$

$$\boldsymbol{ln2 = \frac{1}{2} ln2 \ldots ?}$$

Absolute and conditional convergence

- Given series $\sum_{n=1}^{\infty} a_n$, if $\boldsymbol{\sum_{n=1}^{\infty} |a_n|}$ **converges**

$\sum_{n=1}^{\infty} a_n$ is said to **converge absolutely**.

If $\sum_{n=1}^{\infty} a_n$ converges but $\boldsymbol{\sum_{n=1}^{\infty} |a_n|}$ **does not converge,**

$\sum_{n=1}^{\infty} a_n$ is said to **converge conditionally.**

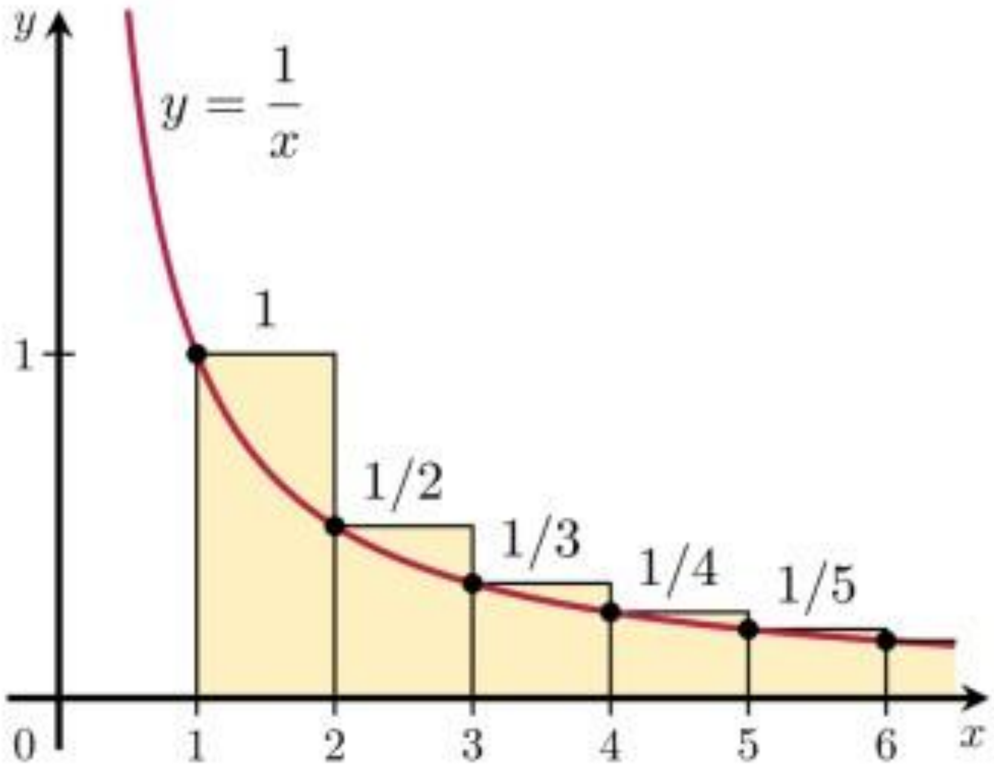
Some tests

- p-series test

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges

if and only if $p > 1$.

Especially, $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

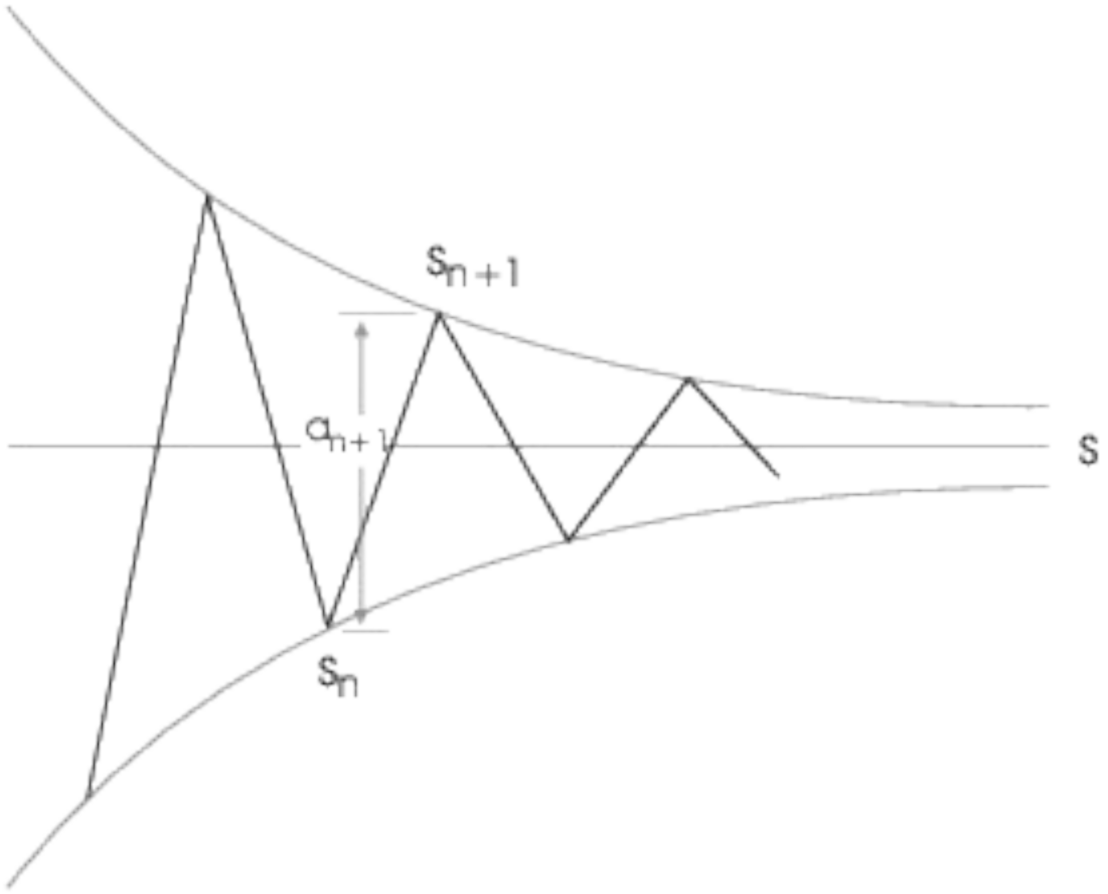


Some tests

- Alternating series test

If a_n is a decreasing sequence of positive real number converging to 0,

$\sum_{n=1}^{\infty}(-1)^n a_n$ converges



Example

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$$\sum_{n=1}^{\infty}(-\frac{1}{2})^n \qquad \sum_{n=1}^{\infty}\frac{1}{2}^n$$

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{n} \qquad \sum_{n=1}^{\infty}\frac{1}{n}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2}$$

Reimann rearrangement theorem

- If a series $\sum_{n=1}^{\infty} a_n$ converge **conditionally**, one can make rearranged series converging to **arbitrary real number.**

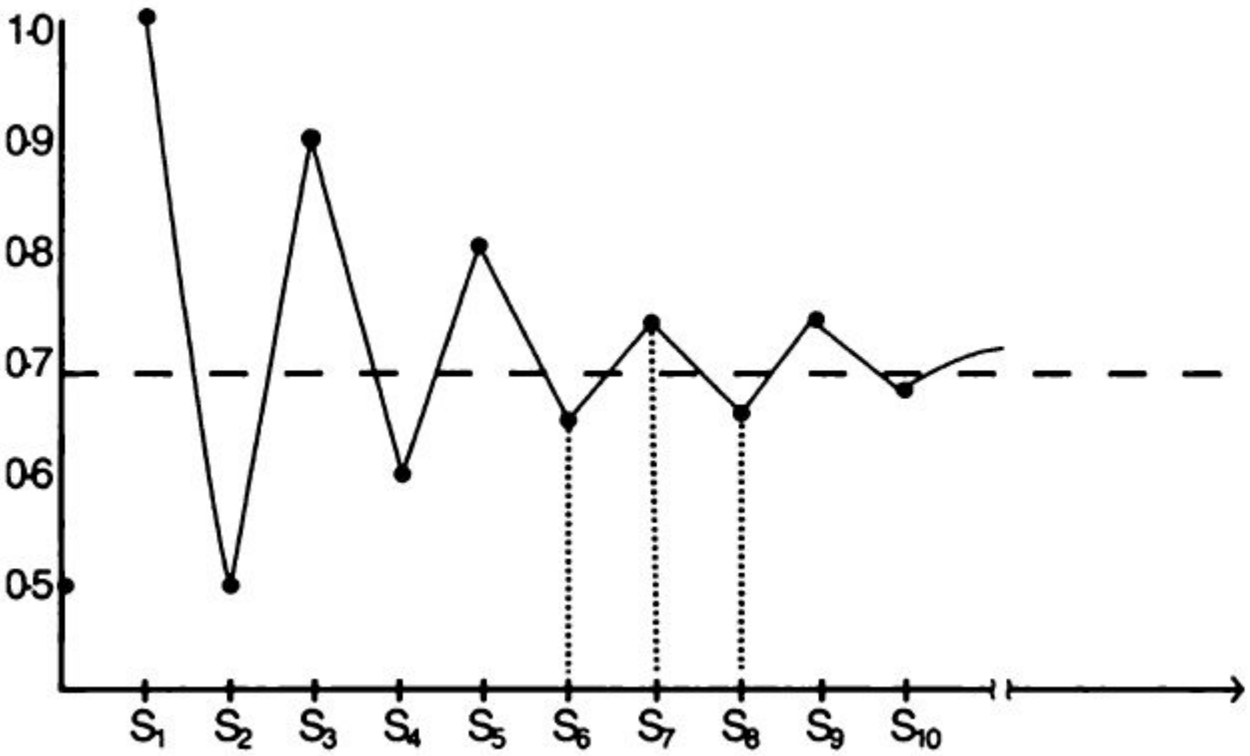
In other words, for any real number M, there exist at least one permutation $\sigma\colon \mathbb{N} \to \mathbb{N}$ such that $\sum_{n=1}^{\infty} a_{\sigma(n)} = M$.

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}\cdots=$$

$$1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}\cdots=$$

원래 번호	1	2	3	4	5	6
바뀐 번호	1	3	4	3	7	5

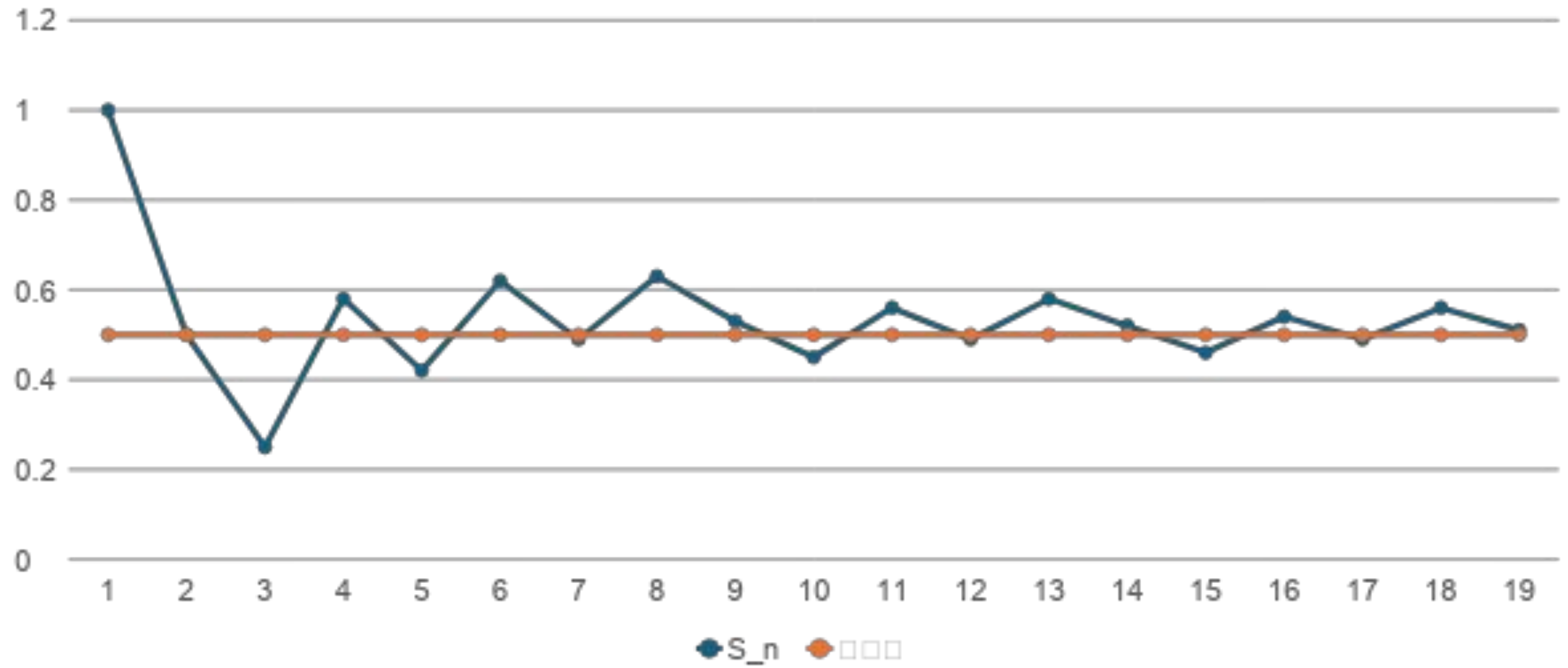
Idea



a_n을 양수와 음수로 나누고
목표치를 넘으면 음수항을 배열
목표치보다 작으면 양수항을 배열

Computation

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
S_n	1	0.5	0.25	0.58	0.42	0.62	0.49	0.63	0.53	0.45	0.56	0.49	0.58	0.52	0.46	0.54	0.49	0.56	0.51



Variation

- If there exist $M \in \mathbb{N}$ for a permutation $\sigma: \mathbb{N} \to \mathbb{N}$

satisfying $|\sigma(n) - n| \leq M$,

following holds.

$$\sum_{n=1}^{\infty} a_{\sigma(n)} = \sum_{n=1}^{\infty} a_n$$