Two perspectives on ‘Satisfaction’

; Topology and Economics

글로벌경제학과 25학번
왕성은

수학 학술동아리 MIMIC 2025. 3. 14

3.14

1592653589793238462643383279502884197169399
3751058209749445923078164062862089986280 34
8253421170679821480865132823066470938446095
5058223172535940812848111745028410270193852 11

파이데이 이벤트

Pi Day Event

- MISSION -

파이를 정확히 삼등분 하라!

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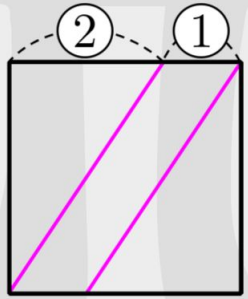
1st

국예성



오예스 - 표준편차 : 0.094281

최소 횟수(2회)로 자르면서,
가장 오차를 최소화 할 수 있는 방식

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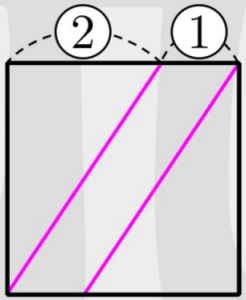

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최소 횟수(2회)로 자르면서,
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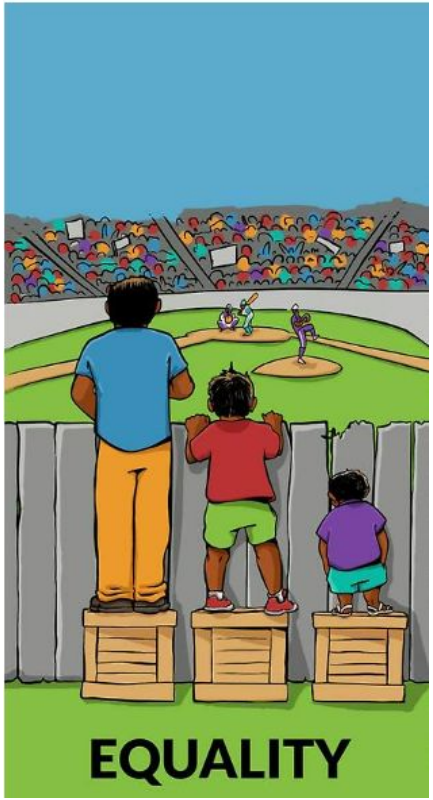
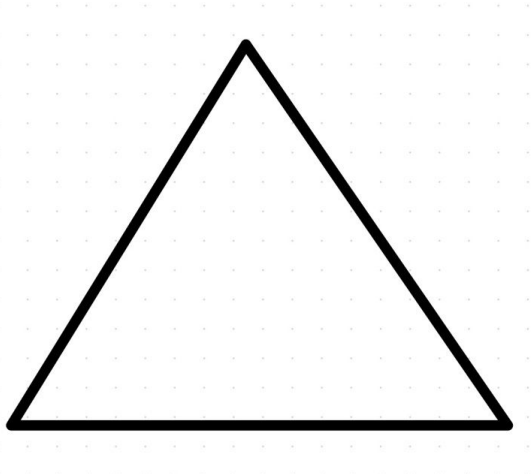
Equality
Distribution

Satisfaction?



Sperner's Lemma
슈페르너의 보조정리



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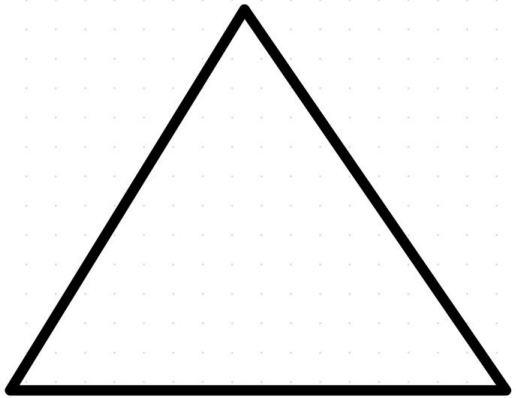


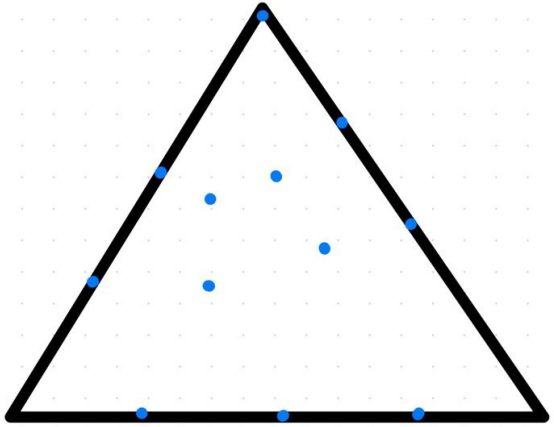
큰 삼각형을 무한히 작은 삼각형으로 쪼개면

어떤 조건을 만족시키는 삼각형이 홀수 개

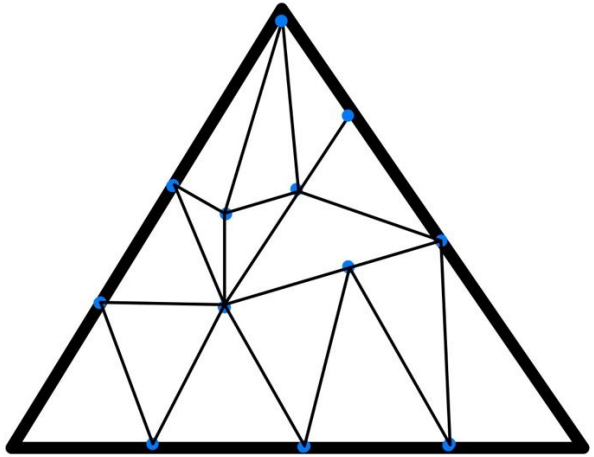
존재한다

If divides the large triangle into infinitely smaller triangles, there are an odd number of triangles(at least one triangles) that satisfied a certain condition.

Sperner's Lemma
슈페르너의 보조정리

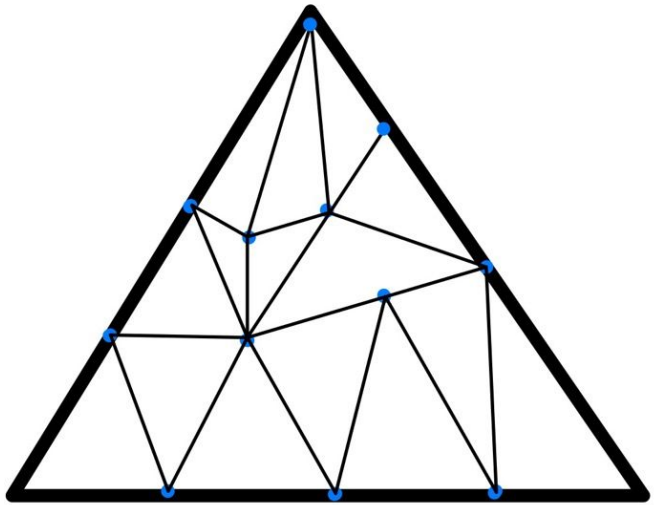


STEP1) 삼각형 내부에 점을 찍는다
(점의 수는 상관 X)



STEP2) 점을 바탕으로 큰 삼각형을
작은 삼각형으로 나눈다

Sperner's Lemma
슈페르너의 보조정리



STEP3) 삼각형의 각 꼭짓점에
고유번호/고유색상을 부여한다

고유번호 : 1~3 중에 택1
고유색상 : 희망하는 색상 3가지 선택

*모든 고유번호/고유색상은 중복 불가

Sperner's Lemma
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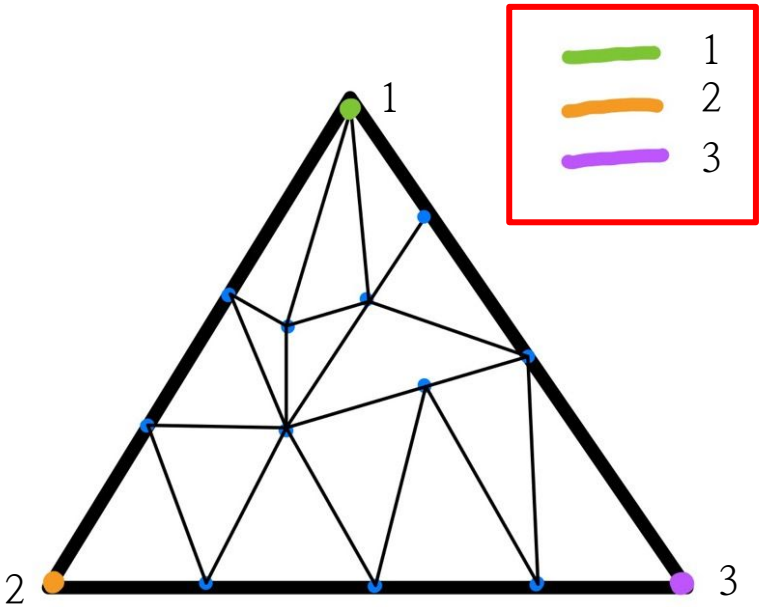

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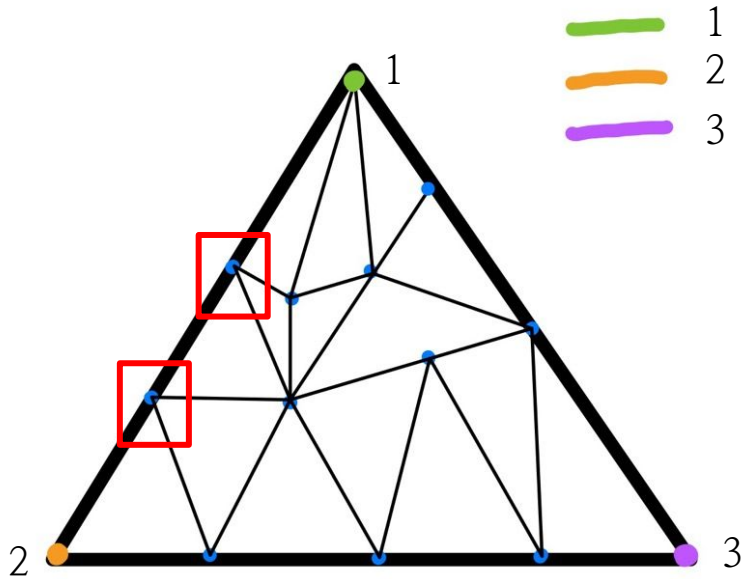

STEP4) 삼각형의 각 변 위의 점에
고유번호/고유색상을 부여한다

조건 : 1-2(G-O)변 위의 점은 1-2(G-O)
의 고유번호/고유색상만
부여할 수 있다.

이 조건은 다른 변에도 동일하게 변형되어
적용된다.

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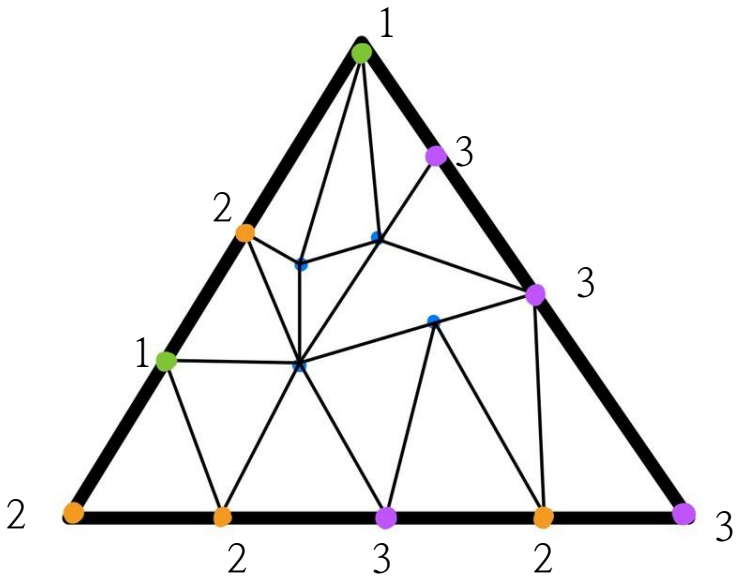
슈페르너의 보조정리



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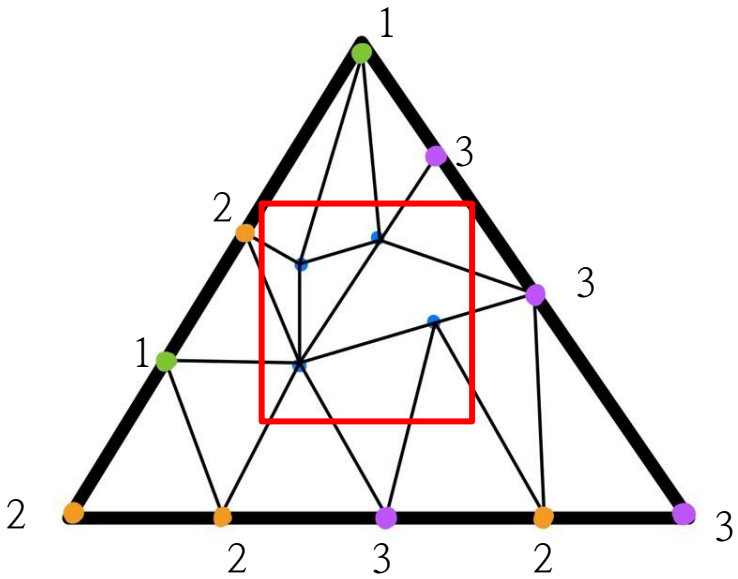
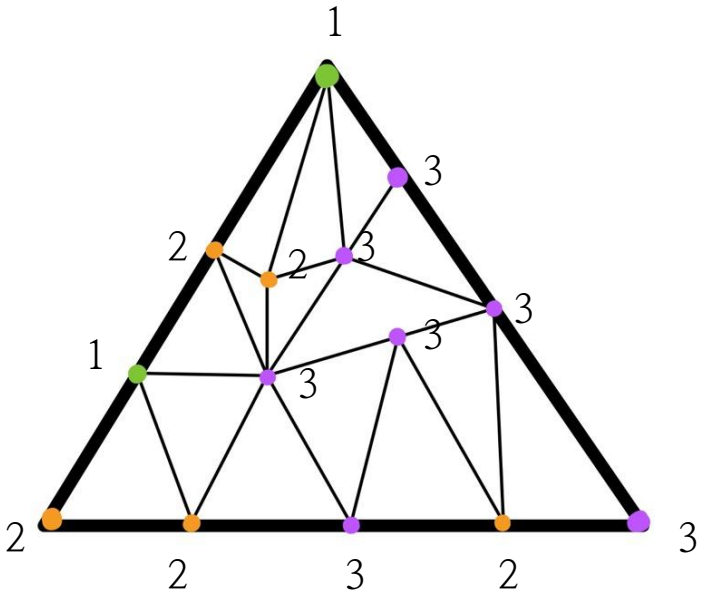


STEP5) 삼각형의 내부의 점에
고유번호/고유색상을 부여한다

조건 : 모든 내부의 점에는 그 어떤
고유번호/고유색상을 부여할 수 있다.

Sperner's Lemma
슈페르너의 보조정리



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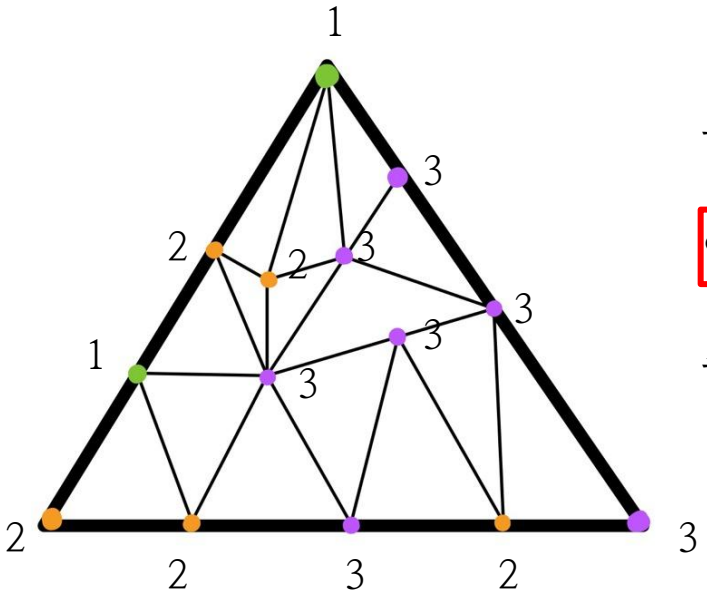
Sperner's Lemma
슈페르너의 보조정리



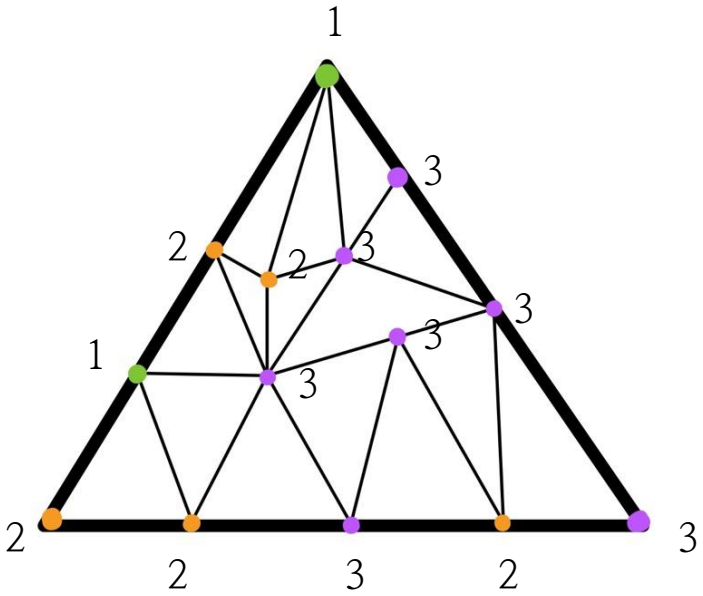
큰 삼각형을 무한히 작은 삼각형으로 쪼개면

어떤 조건을 만족시키는 삼각형이 홀수 개

존재한다

Then, what is certain condition?
Can you guys guess?

Sperner's Lemma
슈페르너의 보조정리



꼭짓점이 1-2-3인 삼각형

(=꼭짓점의 고유색상이 G-O-P인

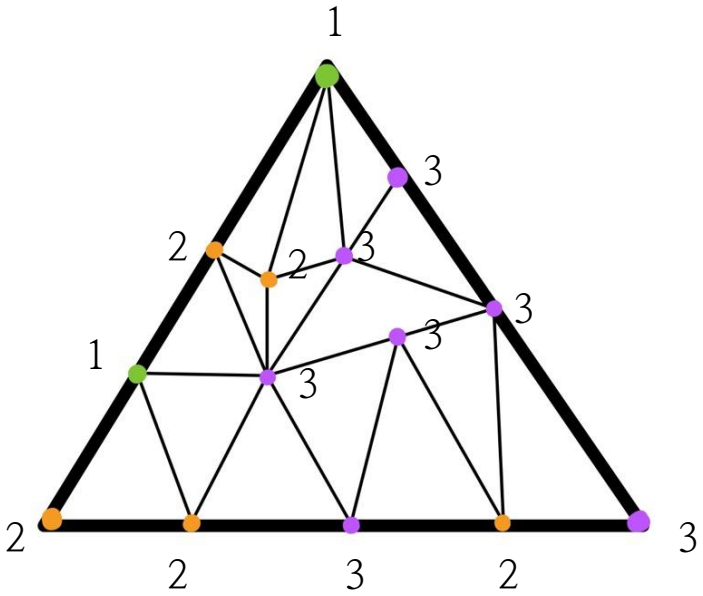
삼각형)이 홀수 개 존재한다

큰 삼각형을 무한히 작은 삼각형으로 쪼개면

위 조건을 만족시키는 삼각형이 홀수 개

존재한다

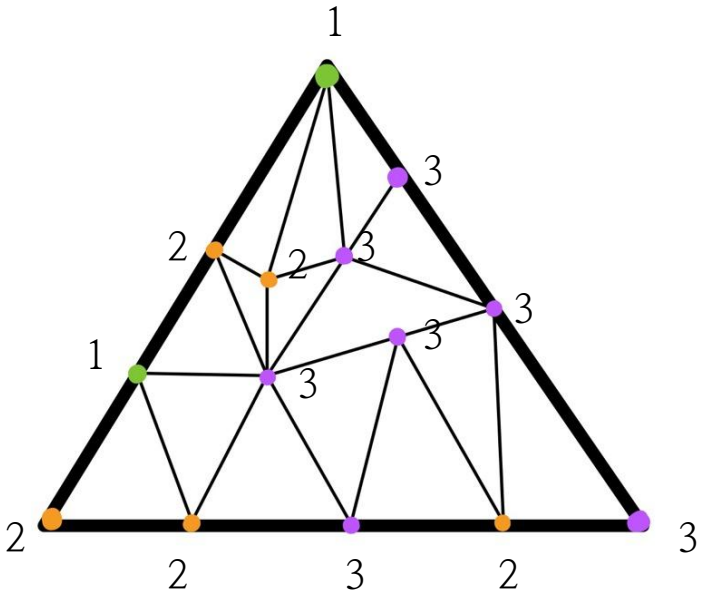
Sperner's Lemma
슈페르너의 보조정리



?

Isn't the triangle you drew the only one that satisfied that condition?

Sperner's Lemma
슈페르너의 보조정리



🙋‍♀️🙋‍♂️VOLUNTEER RECRUITING🙋‍♂️🙋‍♀️

Sperner's Lemma
슈페르너의 보조정리

Certain Condition?

; The number of TRICHROMATIC triangles is odd

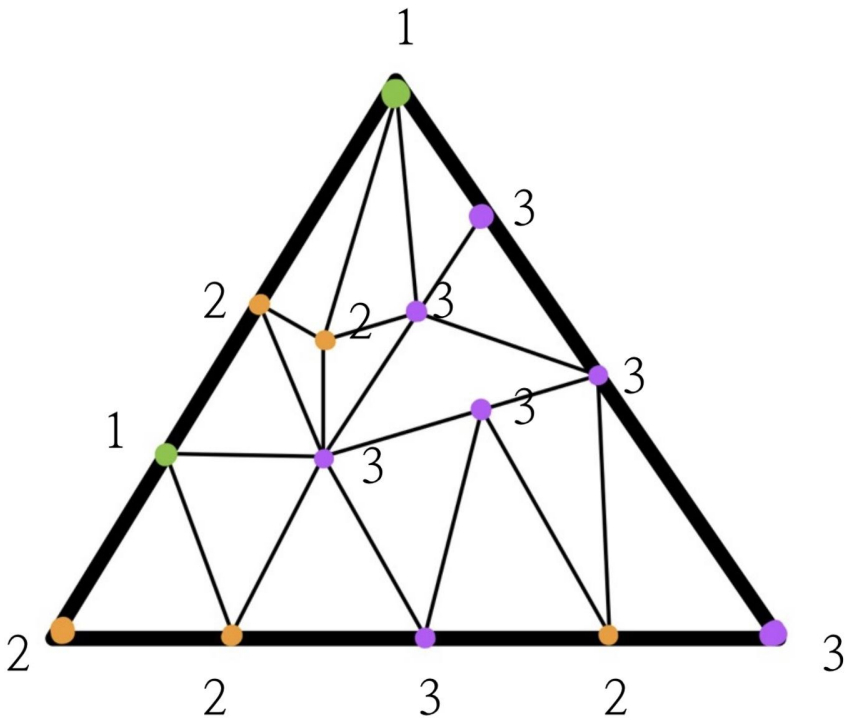
1) utilizing combinatorial method
   ; double counting

2) applying physical method
   ; Kronecker Delta

1) utilizing combinatorial method

; double counting

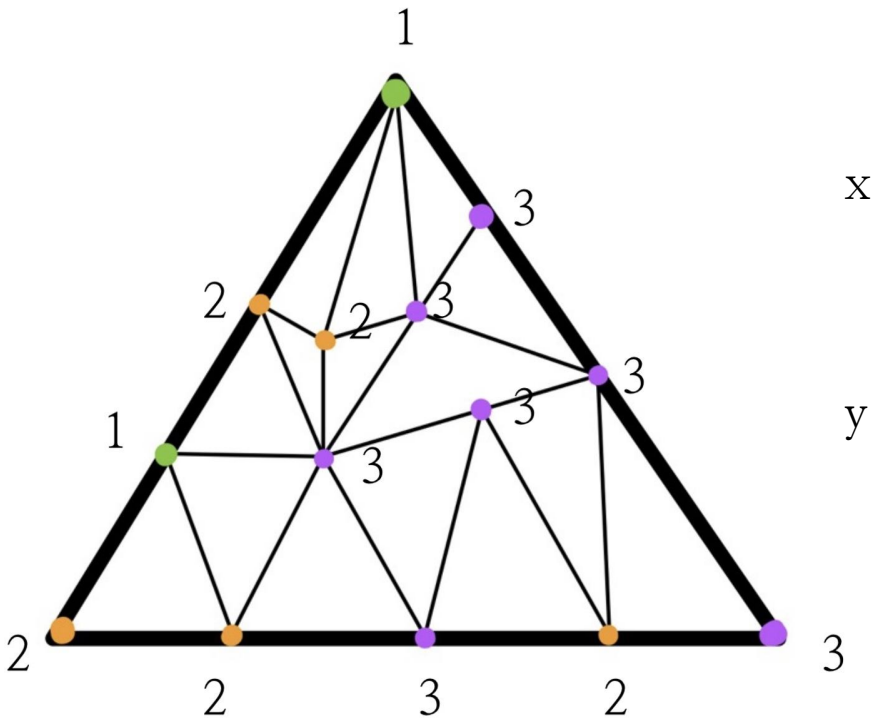
#. Combinatorial method



l : number of trichromatic triangles

k : number of O-P triangles

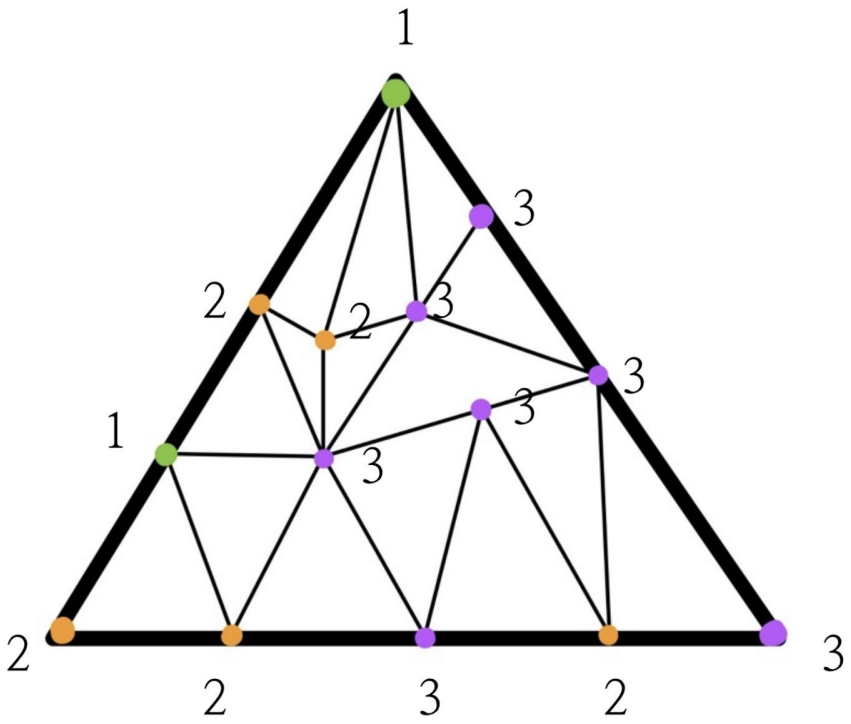
#. Combinatorial method



x : O-P edges on outside

y : O-P edges on inside

#. Combinatorial method



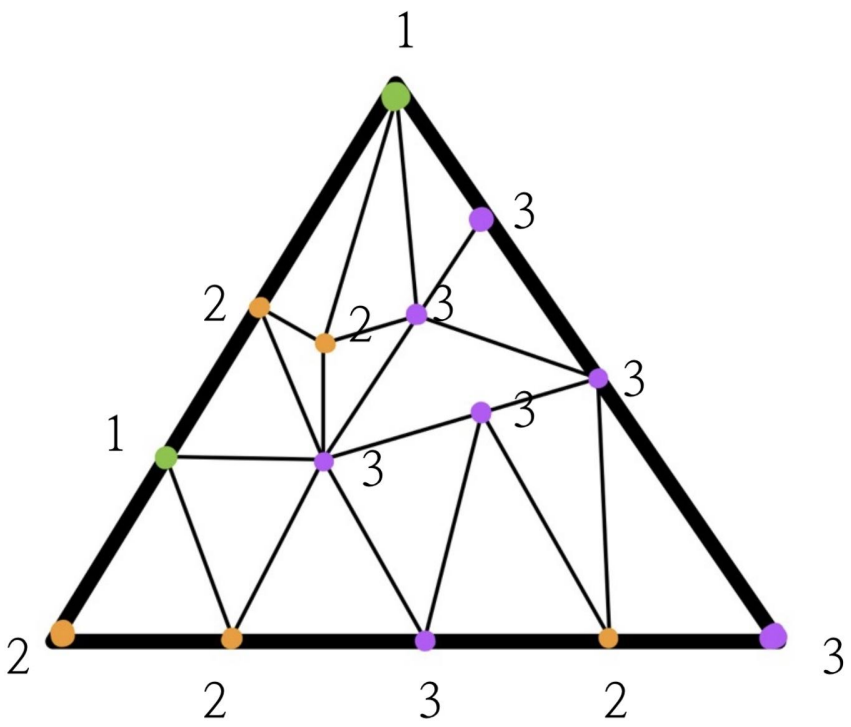
l : number of trichromatic triangles

k : number of O-P triangles

x : O-P edges on outside = odd

y : O-P edges on inside

#. Combinatorial method



l : number of trichromatic triangles

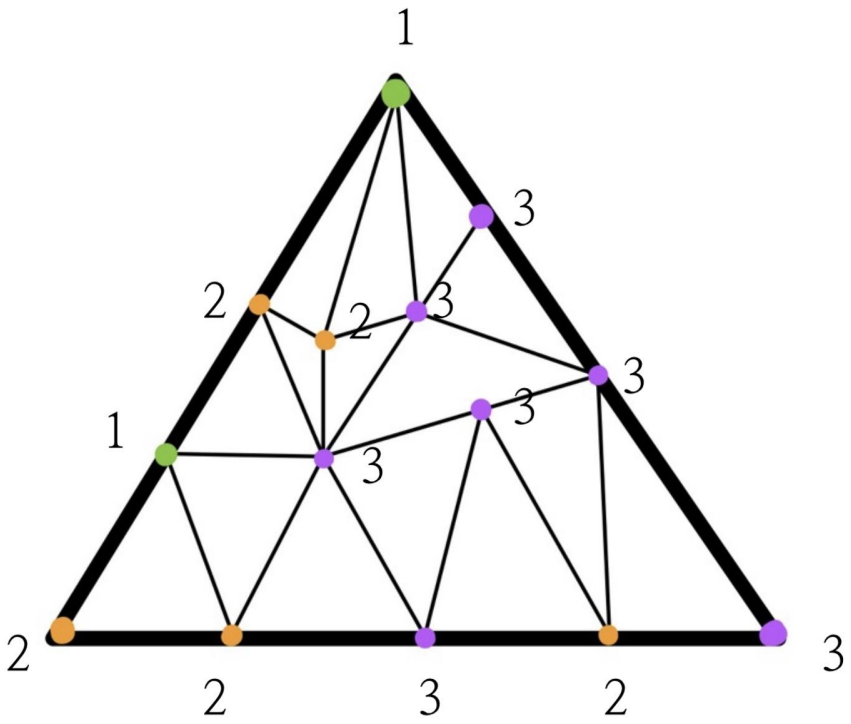
k : number of O-P triangles

x : O-P edges on outside = odd

y : O-P edges on inside

$$l + 2k = x + y$$

#. Combinatorial method



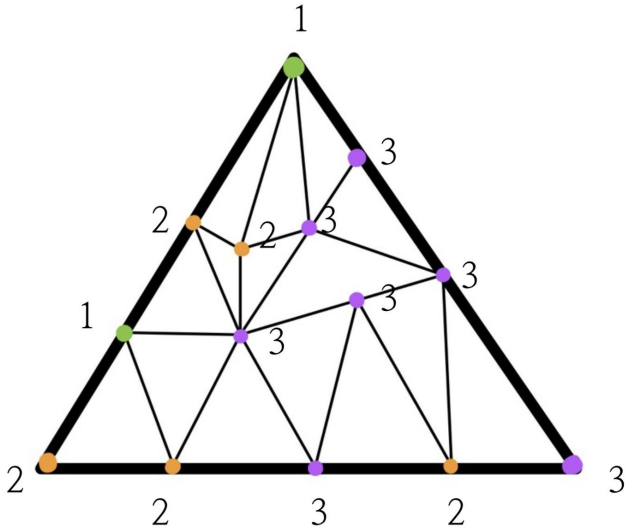
l : number of trichromatic triangles
k : number of O-P triangles

x : O-P edges on outside = odd
y : O-P edges on inside

$$l + 2k = x + y \ (X)$$

=> double counting

#. Combinatorial method



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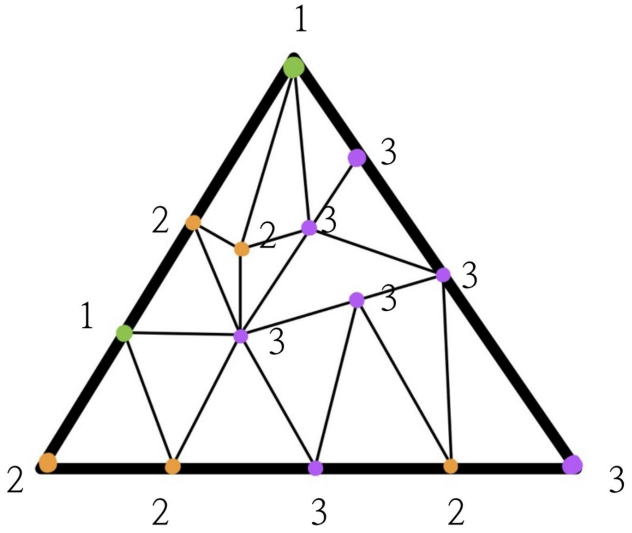


l : number of trichromatic triangles
k : number of O-P triangles

x : O-P edges on outside = odd
y : O-P edges on inside

$$l + 2k = x + 2y$$

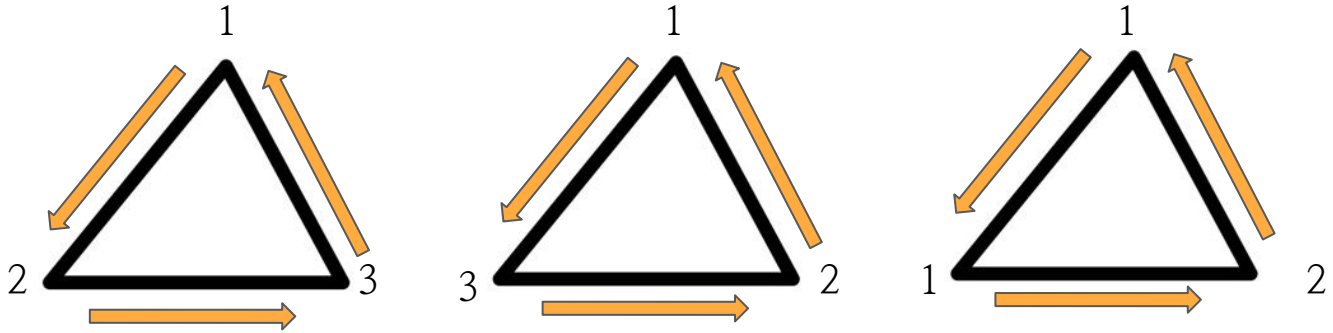
Conclusion : Each valid coloring of the graph results in at least one small triangle that is tri-chromatic. In fact, the number of trichromatic triangles is odd.

2) applying physical method

; path integral

#. Physical method

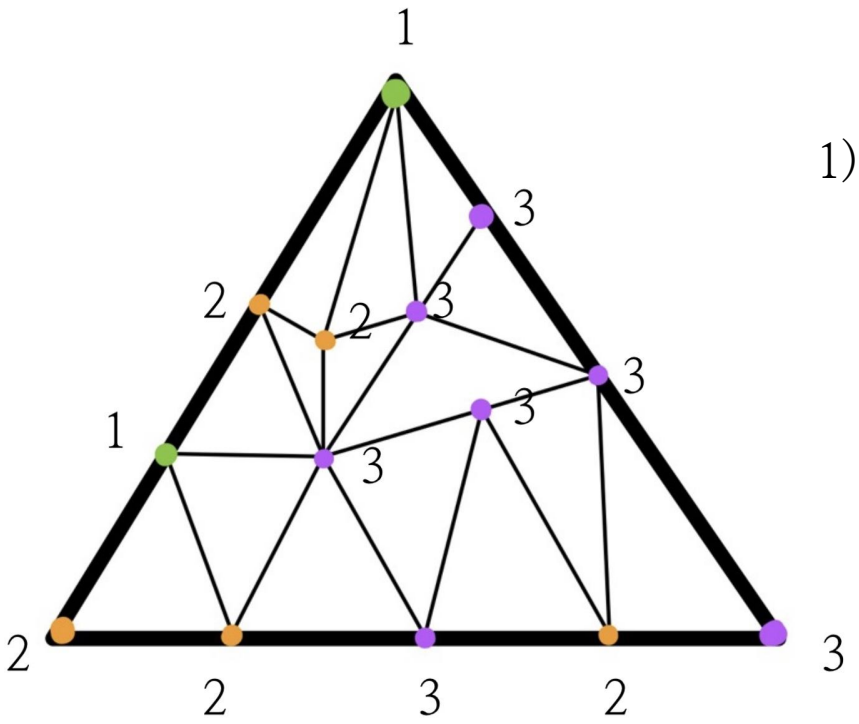
Kronecker Delta(크로네커 델타)



direction of arrow : counterclockwise

1 -> 2 -> 3 direction : +1
3 -> 2 -> 1 direction : -1
1-> 1, 2 -> 2, 3 -> 3 direction: 0

#. Physical method

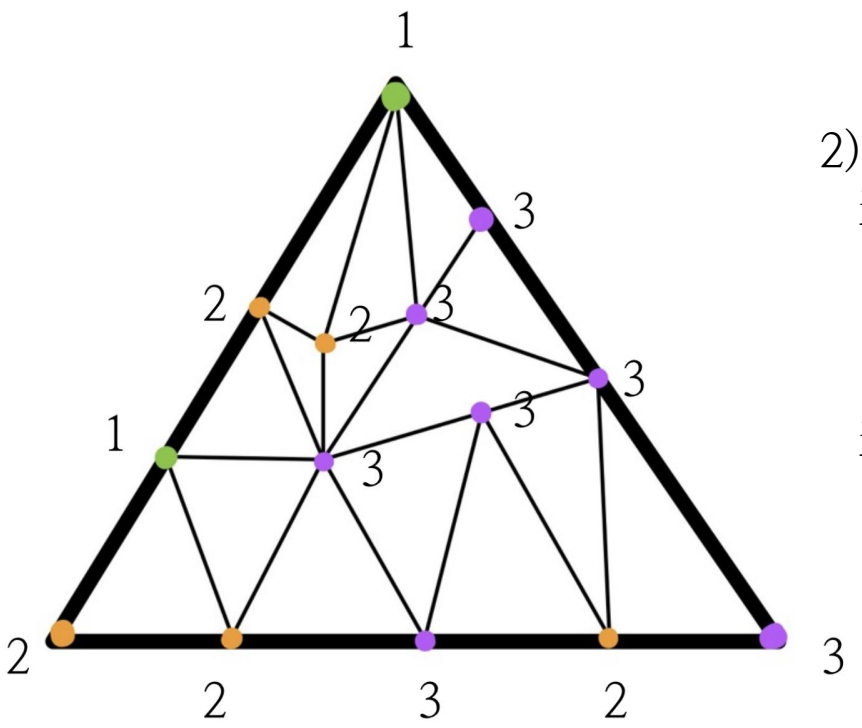


〈Perspective of Edges〉

1) Three edges on big triangle

value = 3

#. Physical method



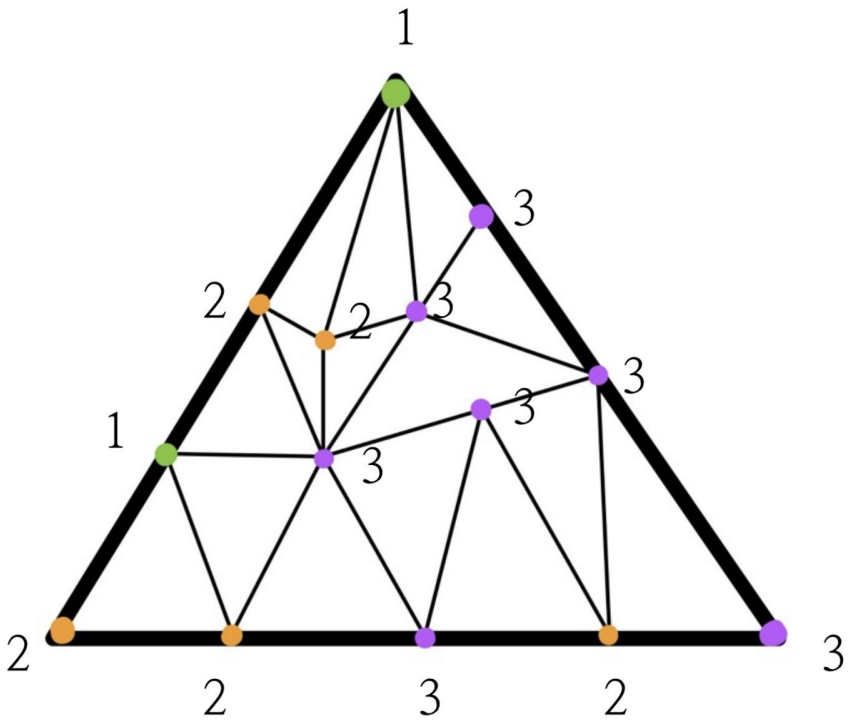
〈Perspective of Edges〉

2) Other edges in big triangle
i) same number on one edge

ii) different numbers on one edge

value = 0

#. Physical method



〈Perspective of Edges〉

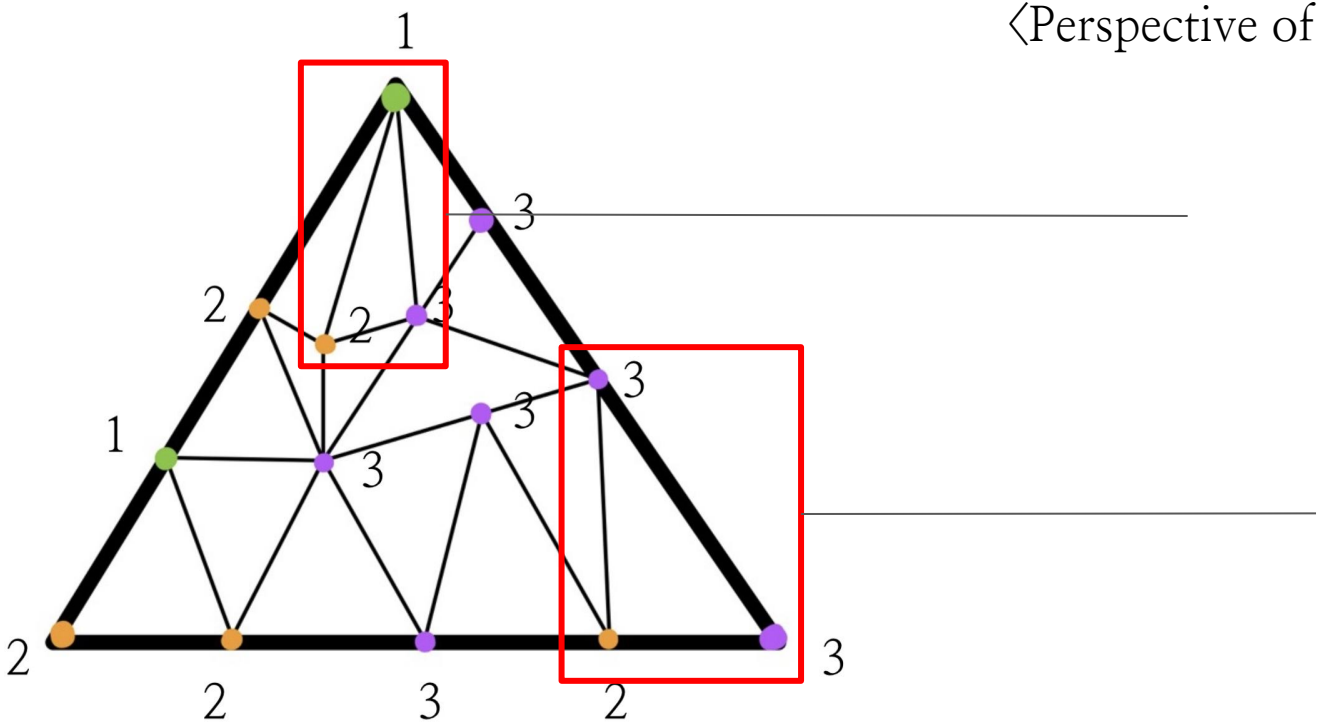
1) Three edges on big triangle
value = 3

2) Other edges in big triangle
value = 0

Total value = 3

#. Physical method

〈Perspective of Triangle〉



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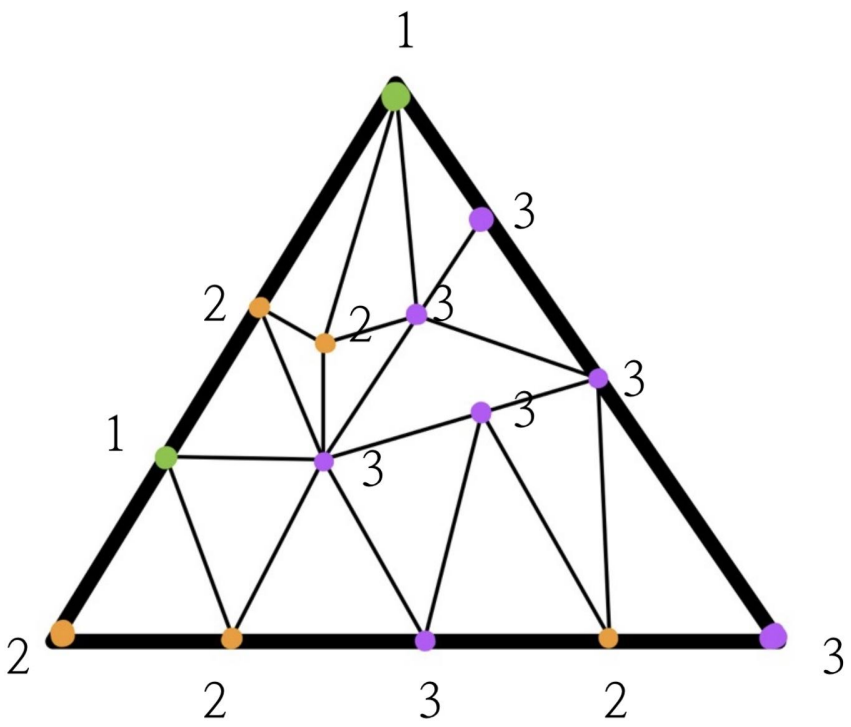

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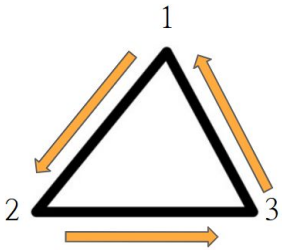
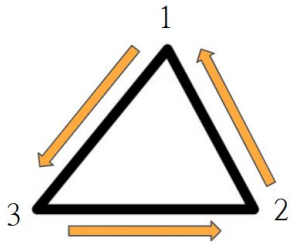


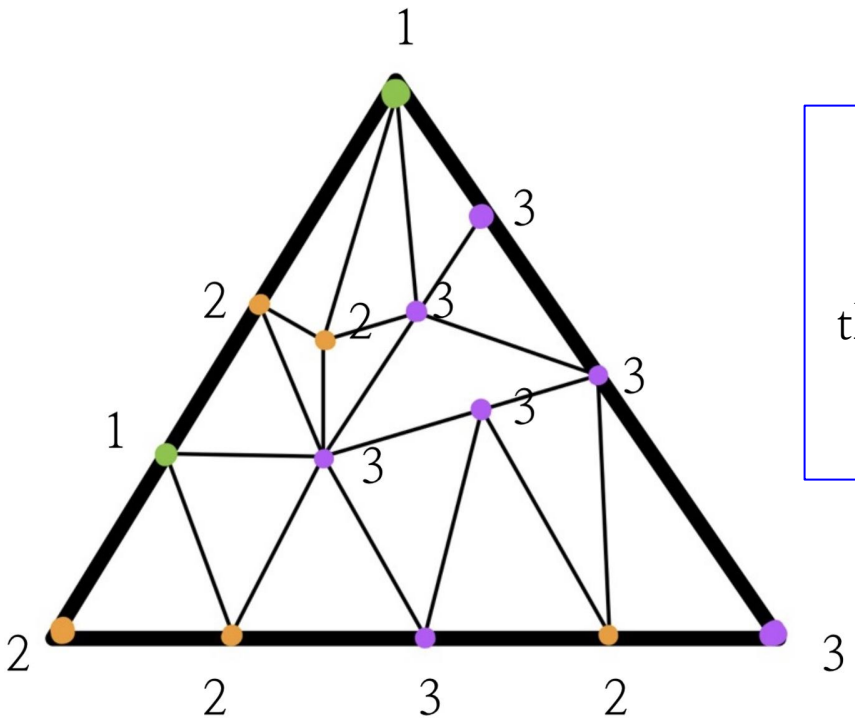
+3



−3

two values' sum = 3

#. Physical method



Conclusion：Each valid coloring of the graph results in at least one small triangle that is tri-chromatic. In fact, the number of trichromatic triangles is odd.


나
너무 많은
일이 있었어
힘들다진짜

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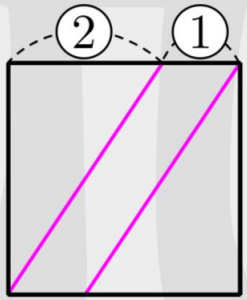
1st

국예성

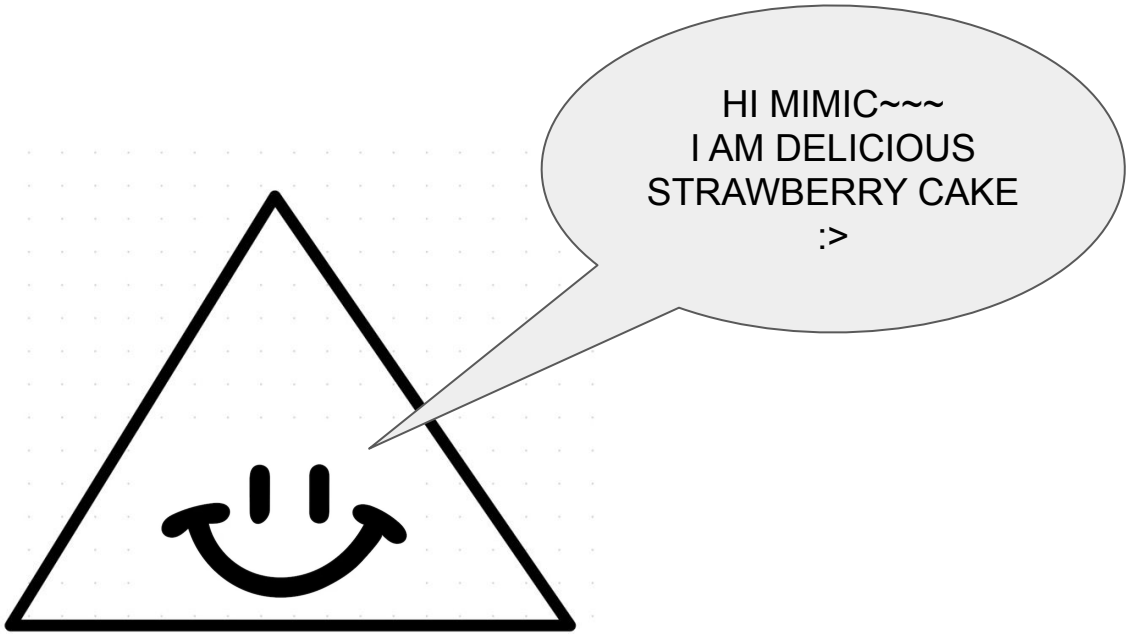


오예스 - 표준편차 : 0.094281

최소 횟수(2회)로 자르면서,
가장 오차를 최소화 할 수 있는 방식

How can we cut the cake to satisfy everyone?

#. Cutting Cake



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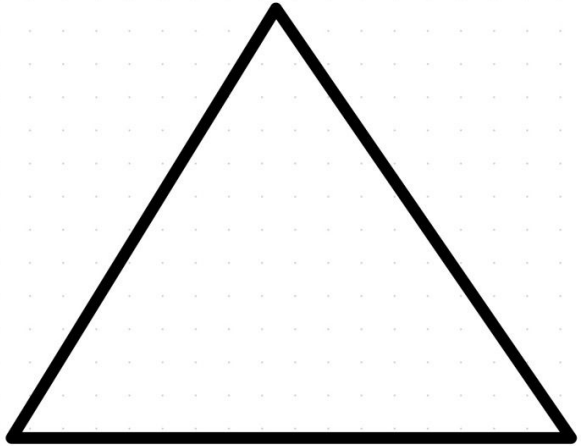
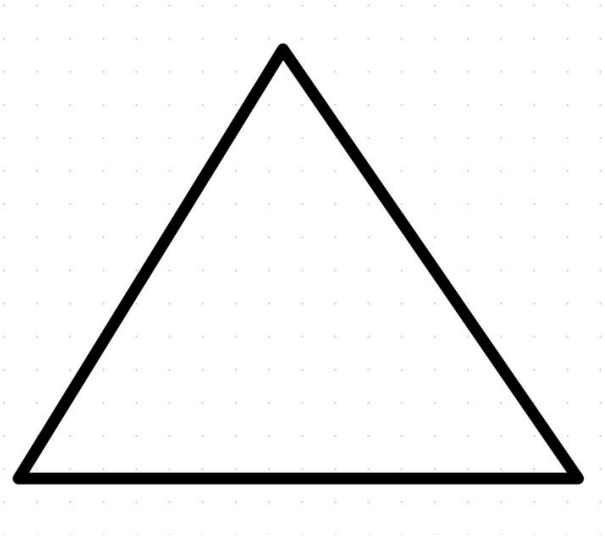


STEP 1) Pick any one spot that make tripartition cake

STEP 2) Give three pieces of cake to three different people

STEP 3) Find out who is satisfied at that split point
(there is at least one person who satisfied)

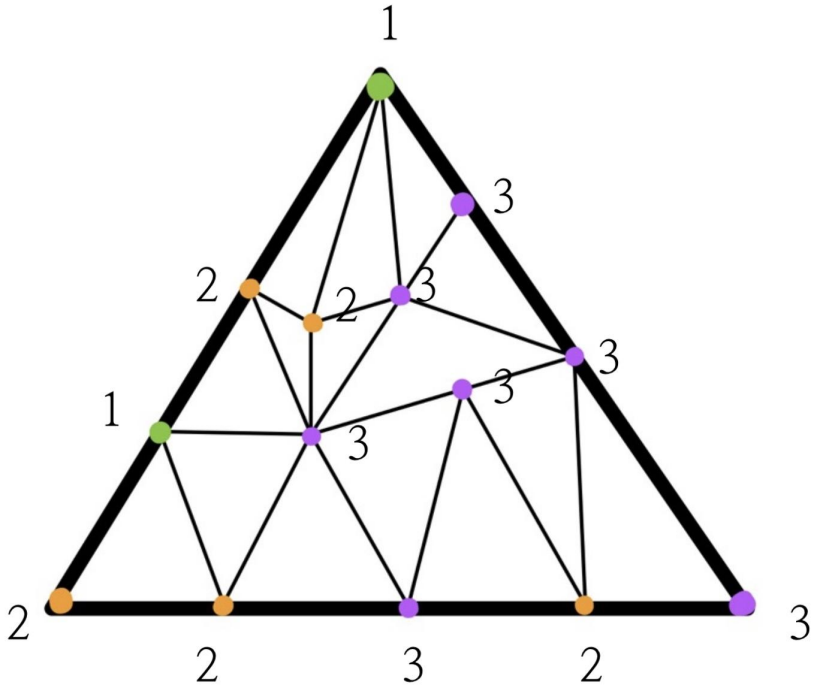
#. Cutting Cake



STEP 4) Check that split point on cake with the number of person who satisfied at that split point

STEP 5) Find many split points as you can do and repeat the procedure

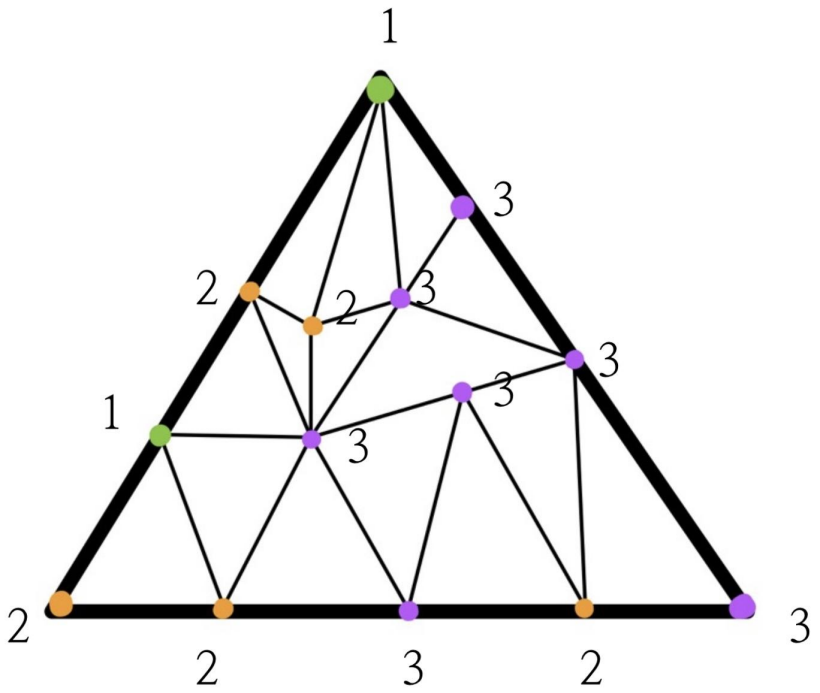
#. Cutting Cake



STEP 6) Pick one trichromatic triangle
(We don't care the place of trichromatic triangle.)

STEP 7) Repeat the procedure

#. Cutting Cake



STEP 8) Repeat the procedure

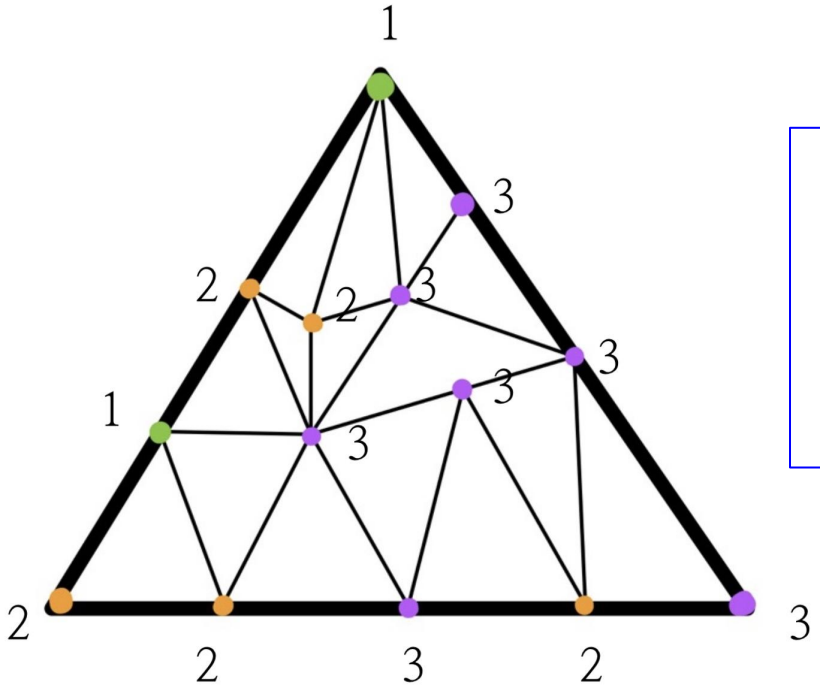
STEP 9) Repeat the procedure

STEP 10) Repeat the procedure

.
.
.

STEP 12345678910) Repeat the procedure

#. Cutting Cake



The triangle converge to a point, and that point becomes the split point that satisfies everyone(who will eat that cake)

수학 :
모두 만족 시키게
나눠줄게 ㅇㅇ

경제학 :
반론할게 ㅇㅇ

#. Economics's refute

첫번째 케이크였다면? 모두가 만족했을 것!

#. Economics's refute

#. Law of diminishing marginal utility

한계 효용 체감의 법칙

#. Law of diminishing marginal utility

한계 효용 체감의 법칙

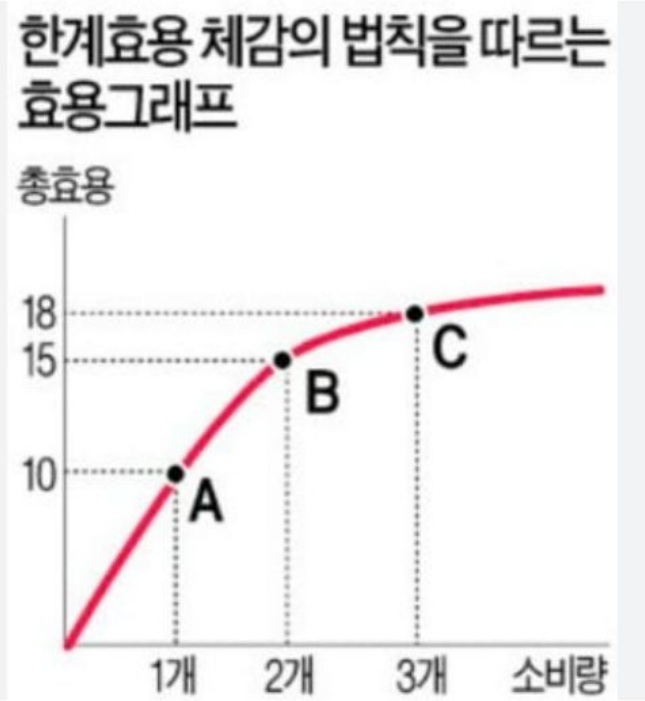
additional

; 어떤 재화나 서비스의 소비량/생산량을 조금 더 늘리는 경우 비용/효용수준이 추가적으로 얼마나 증가하는가를 측정한 것

#. Law of diminishing marginal utility

한계 효용 체감의 법칙

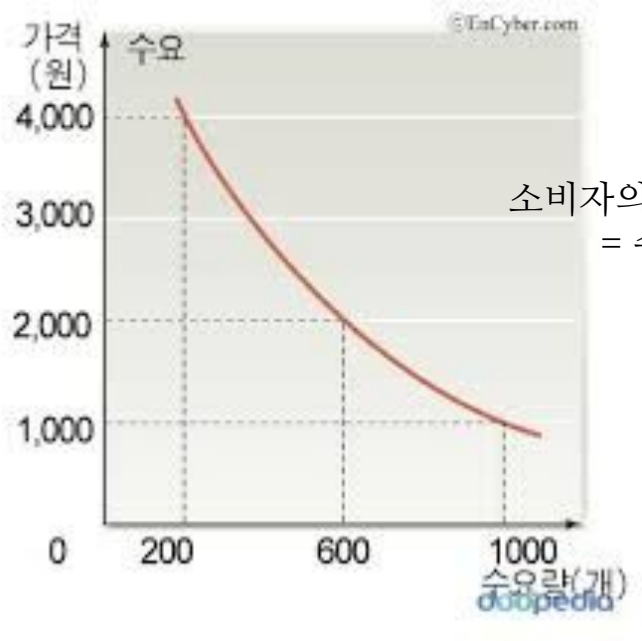
1) JUST 효용



#. Law of diminishing marginal utility

한계 효용 체감의 법칙

2) 잉여(Surplus) = 효용



소비자의 지불 용의 – 실제로 지불한 금액
= 수요곡선의 아랫부분과 가격 수준
윗부분의 면적

#. Law of diminishing marginal utility

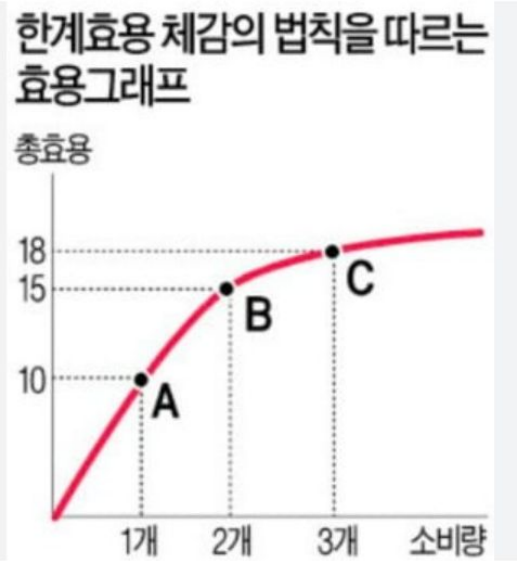
한계 효용 체감의 법칙

decrease

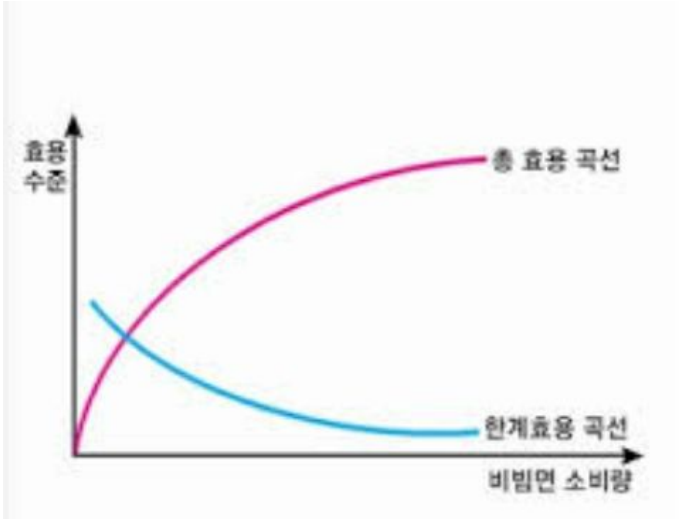
#. Law of diminishing marginal utility

한계 효용 체감의 법칙

〈총효용 그래프〉



〈한계효용 그래프〉



#. Law of diminishing marginal utility

한계 효용 체감의 법칙 in cutting cake

〈첫번째 케이크〉

〈두번째 케이크〉

#. Law of diminishing marginal utility

한계 효용 체감의 법칙 in cutting cake

Conclusion : Because of Law of diminishing marginal utility, the point at which the cake is divided mathematically may not actually be the split point at which everyone is satisfied.

So, we have to consider individual's UTILITY.

Feel free to ask question :〉

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왕성은