Invariant of knot

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매듭의 정의

- 어떤걸 정의하느냐 현실에 있는 매듭을 표현 하고자 함

  ex) 신발끈, DNA, 끈 이론 등...

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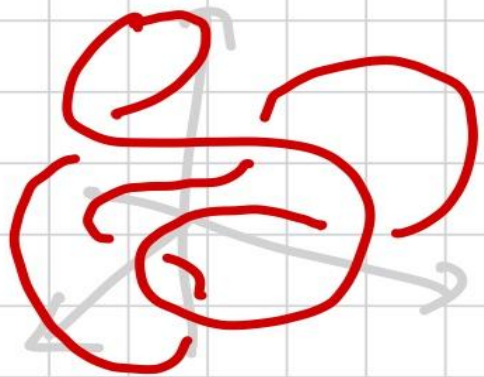
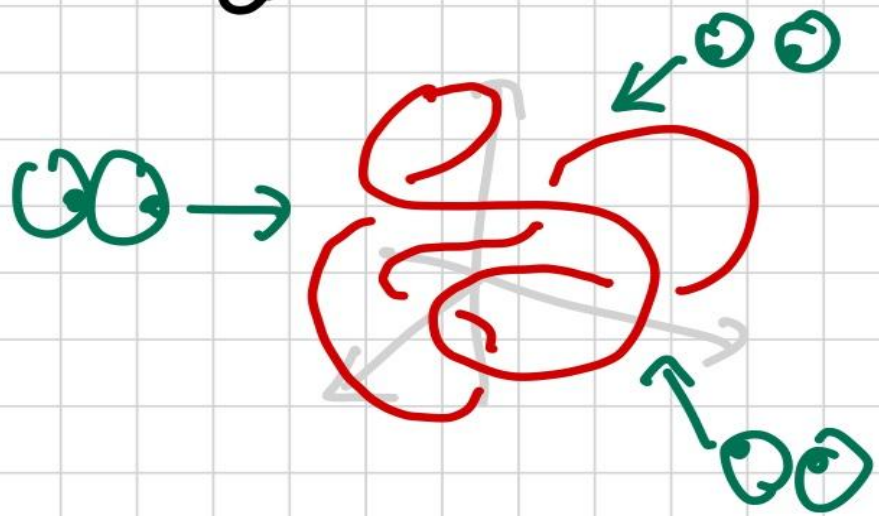
→ 3차원에 있는 시작과 끝이 이어진 하나의 기다란 선



: Knot.

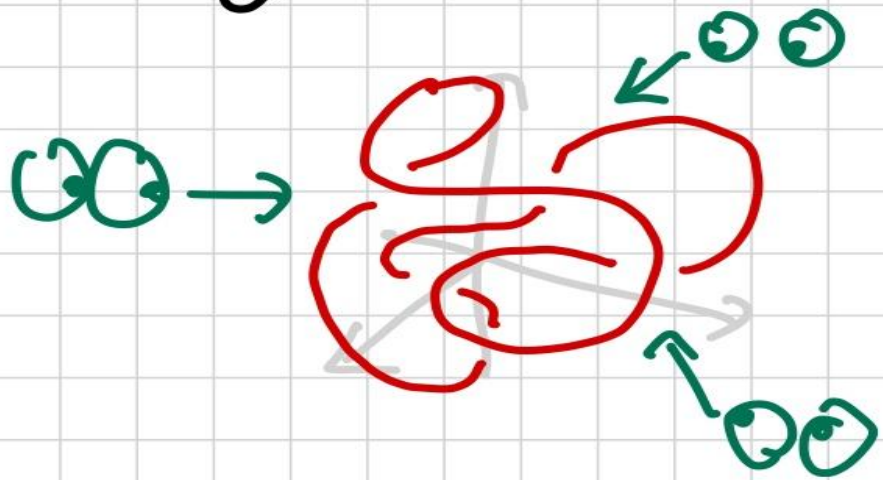
Knot diagram 은 ⇒ 하나의 사진을 보는 것



같은 Knot 이여도

diagram은 다를 수 있다!

Knot diagram 은 => 하나의 사진을 보는 것



같은 Knot 이여도
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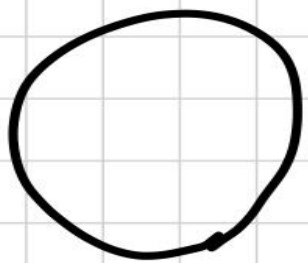
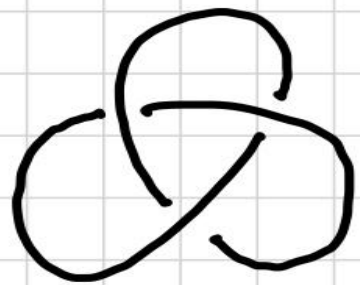
Knot diagram 예시



unknot



trefoil knot(3_1)

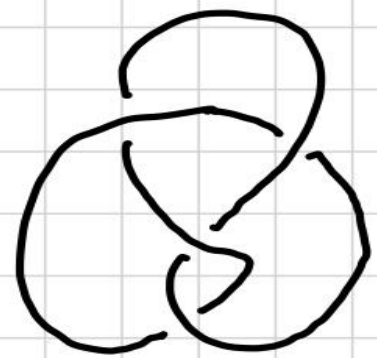


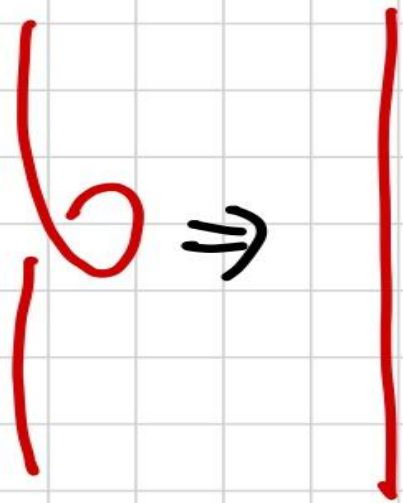
Figure-8(4_1)

⇒ diagram만 보고서 어떻게 같은 knot인지 확인하지?

⇒ Reidemeister move 사용.

What is Reidemeister move?

Type 1



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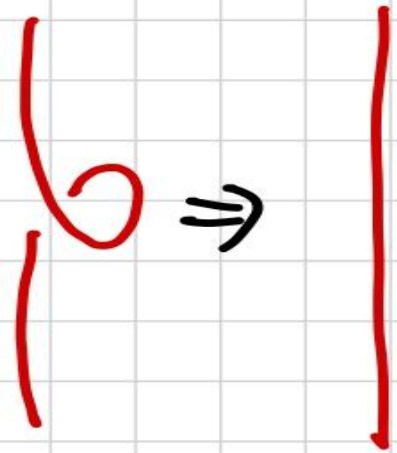
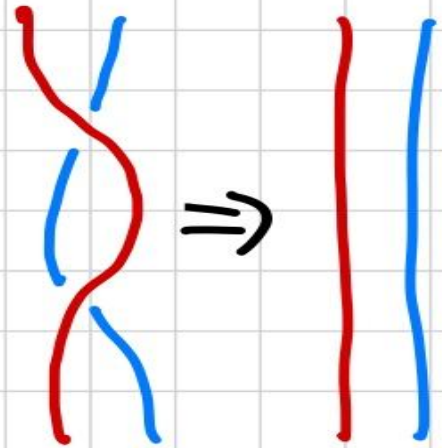


Type 2



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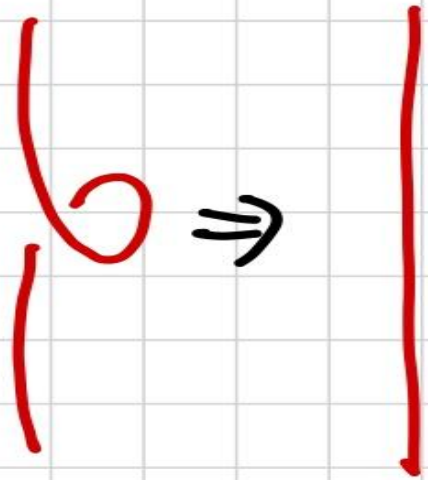
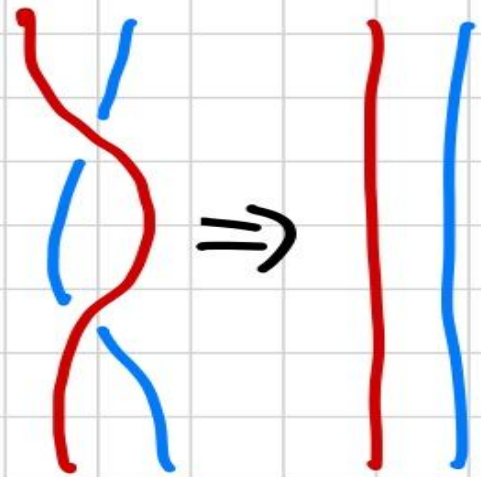
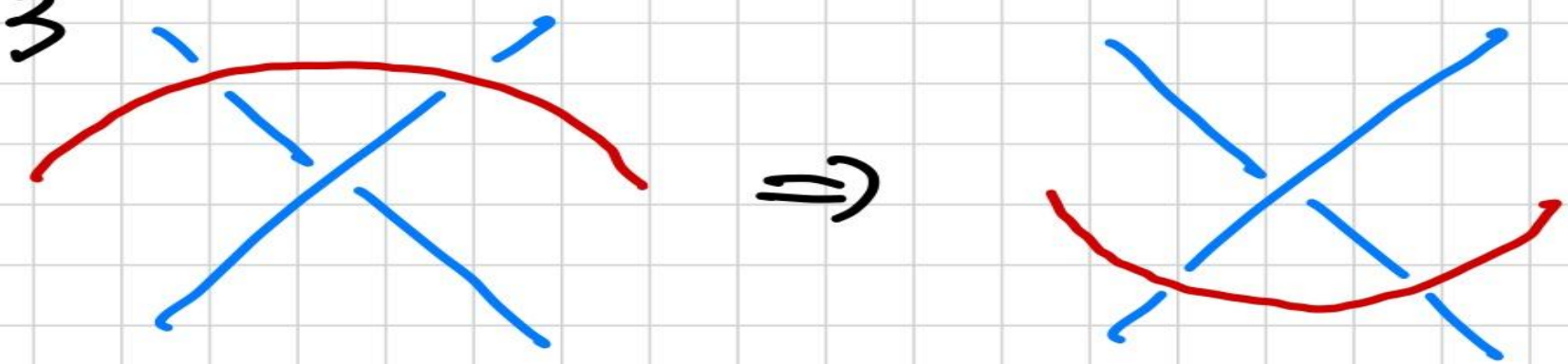


Type 2



Type 3



Reidemeister move 1, 2, 3를 해도
모두 같은 knot 이다!

두 매듭이 같은지 다른지 어떻게 증명할까?

두 맵들이 같은지 다른지 어떻게 증명할?

⇒ Invariant 확인

두 매듭이 같은지 다른지 어떻게 증명할?

⇒ Invariant 확인

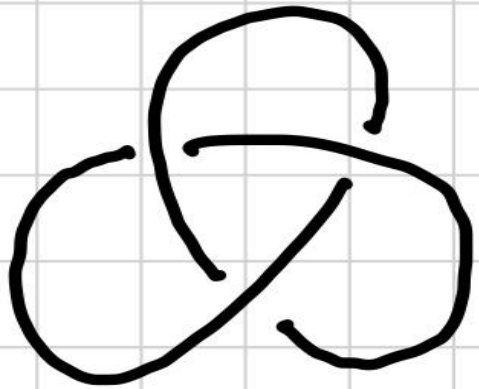
① Alternating - ez

② Jones polynomial - hard

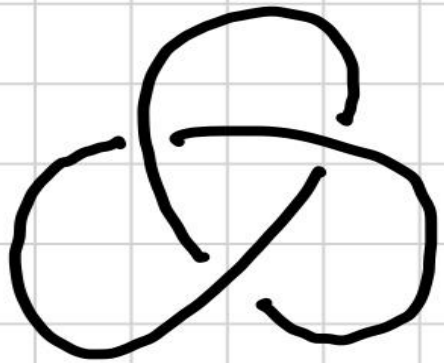
③ Turaev / Seifert genus - really hard

Alternating

- 매듭의 선을 따라갈때 위-아래-위-아래 바뀌면서 가는지 여부.



: alternating knot



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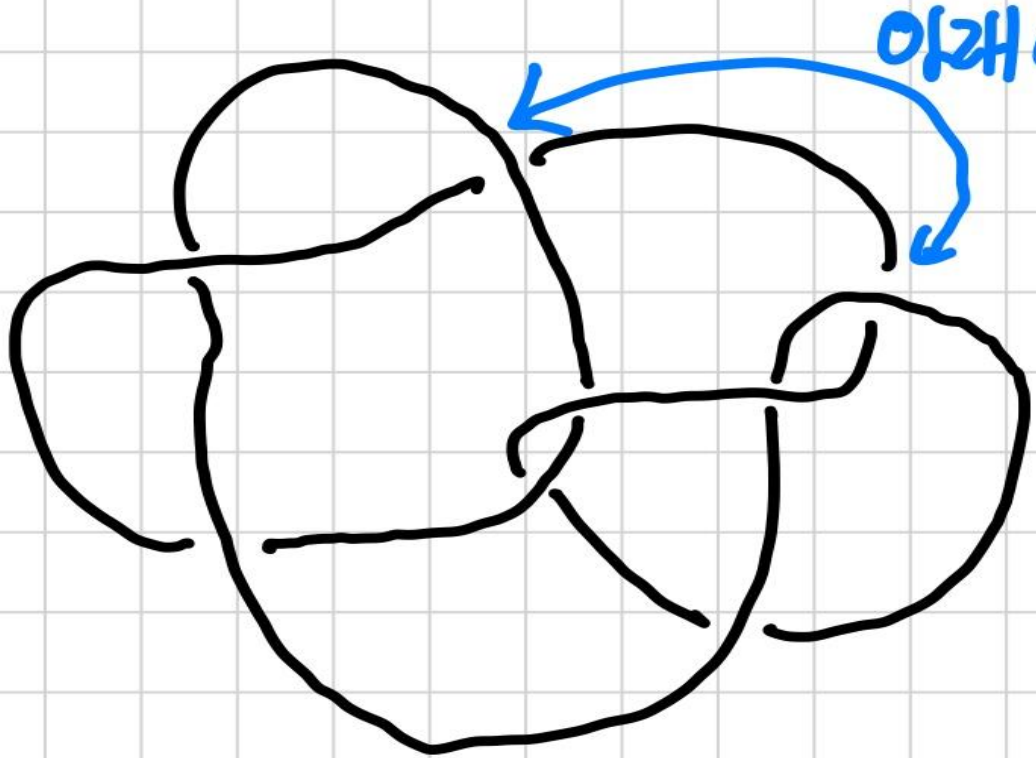


: non-alternating

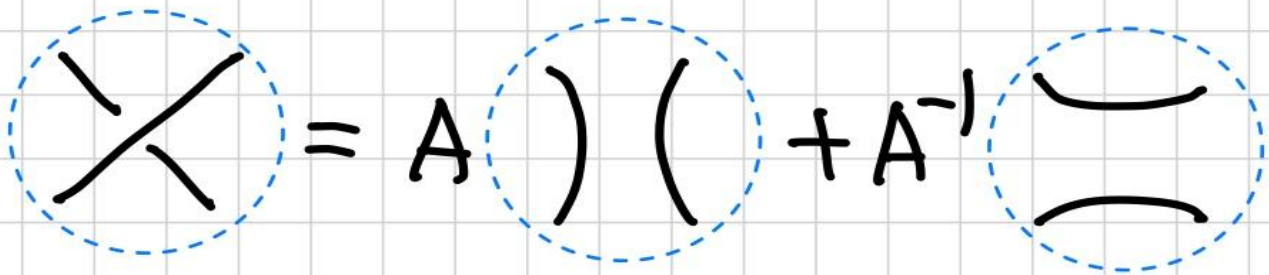
Jones polynomial

- 정의 - Braid group에서 시작해서... (생략)

계산 방법 -

$X[L] = (-A^3)^{-w(L)} \langle L \rangle$, where $\langle L \rangle$ can be calculated by 3 rules

① $\langle O \rangle = 1$ (O is unknot)

② 

③ $\langle O \sqcup L \rangle = (-A^2 - A^{-2}) \langle L \rangle$

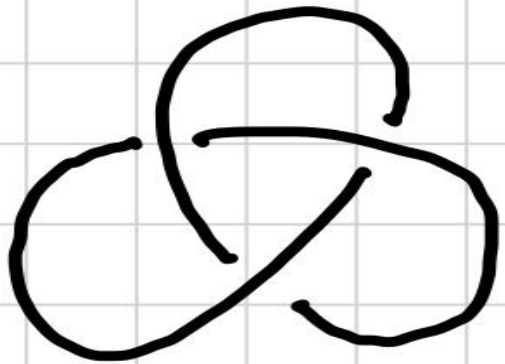
Time complexity $\leq O(2^n)$ — 너무 느림!

↓ 줄이는 알고리즘 존재

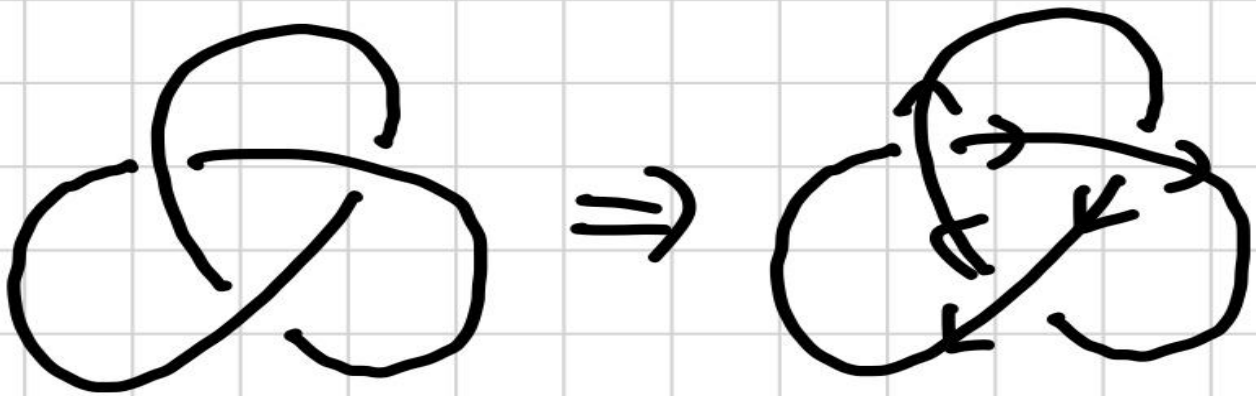
$\leq O(2^{\frac{n}{p}})$

↑ 복잡한 무언가

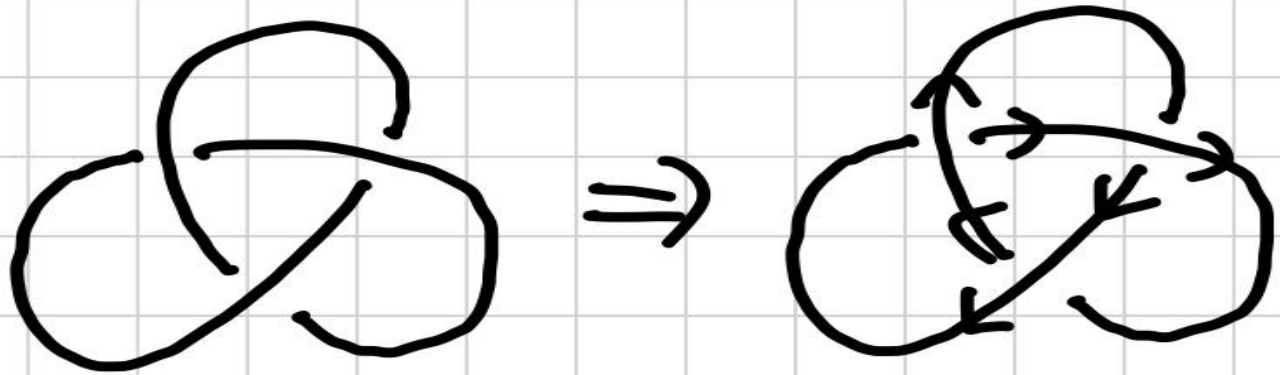
Seifert surface

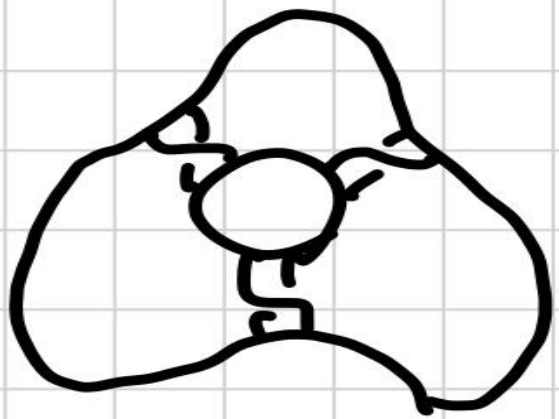


Seifert surface

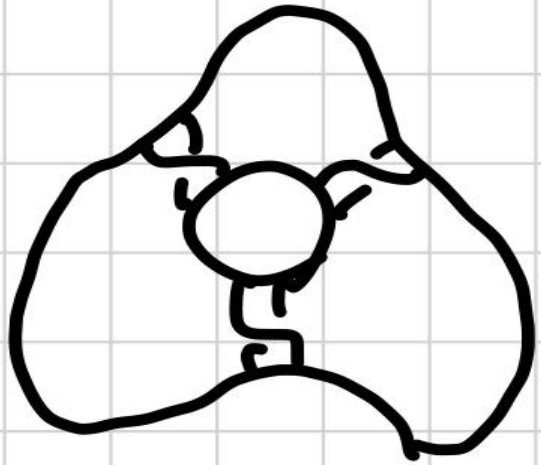


Seifert surface





: Seifert surface



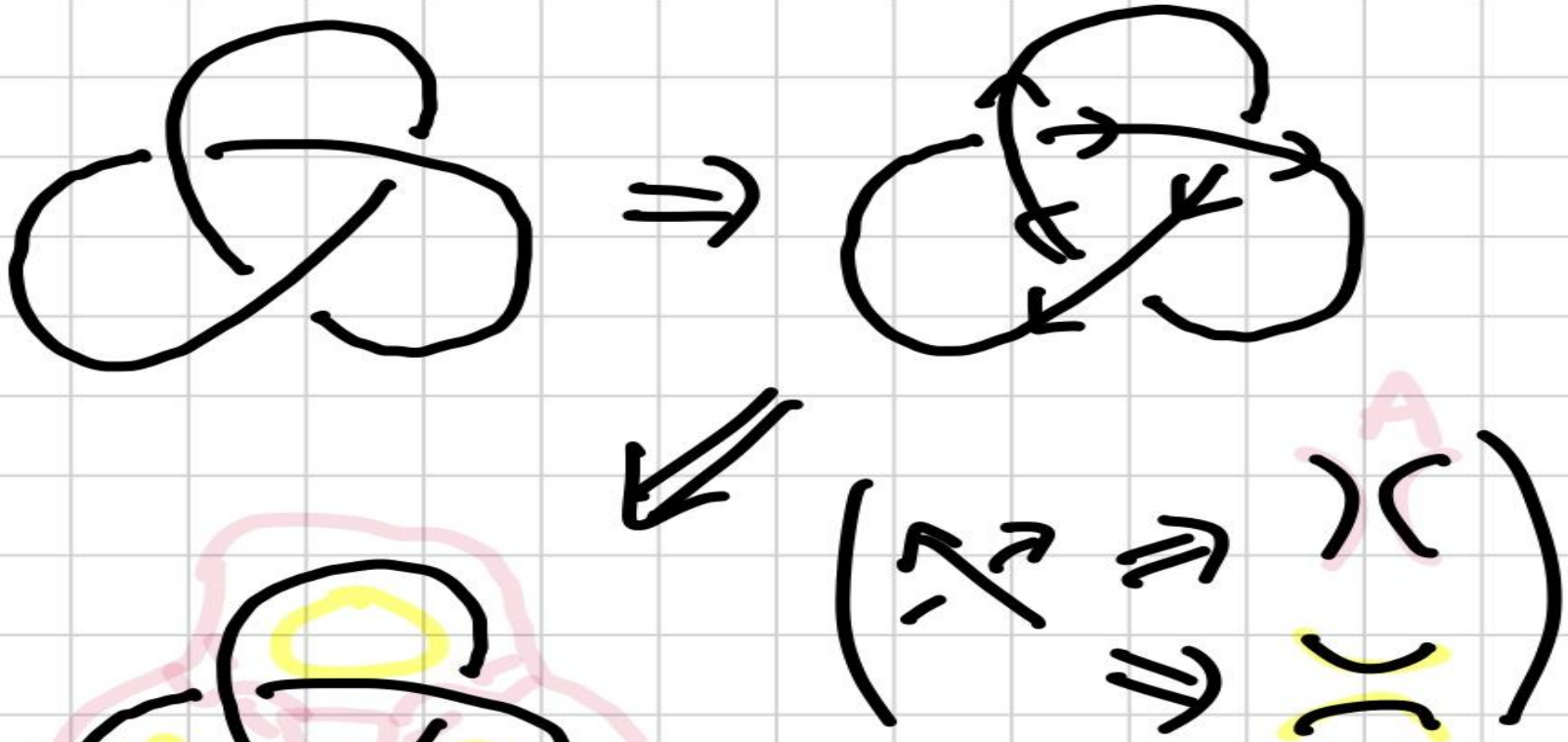
: seifert surface
oriented surface.

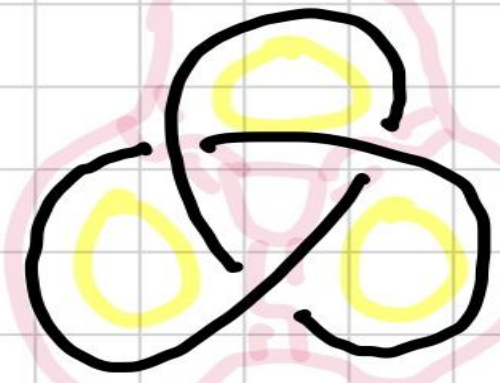
genus 계산 : euler characteristic 사용

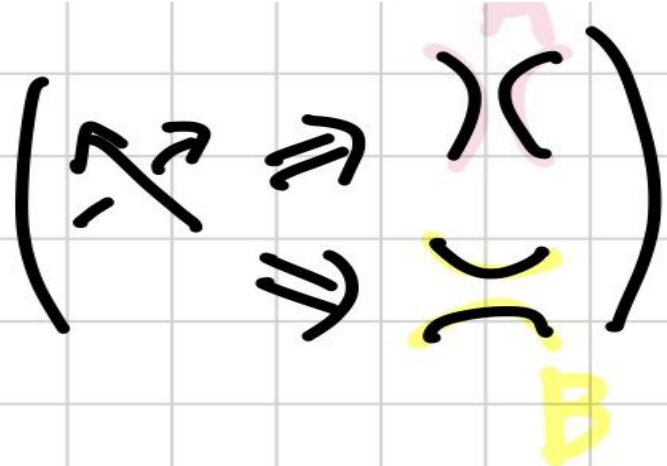
$g_S = \frac{1}{2}(c - s + 1)$ 중에서 최소

c: 교차수, s: 원 개수

Turaev Surface







$$Q_T = \frac{1}{2}(C - S_A - S_B)$$

⇒ 일반적으로 $Q_T \leq Q_S$

Problem

① Jones poly. 계산량 줄이기

② Q_T와 Q_S의 관계 찾기